Comparaison of Several Estimation Procedures for Long Term Behavior

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Comparaison of Several Estimation Procedures for Long Term Behavior

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Abstract

In this paper, nine memory parameter estimation procedures for the fractionally integrated I(d) process, semi-parametric and parametric, which prevail in the existing literature are reviewed; through the simulation study under the ARFIMA(p, d, q) setting we cast a light on the finite sample performance of these estimation procedures for the non-stationary long memory time series. As a by-product of this study, we provide a bandwidth parameter selection strategy for the frequency domain estimation and an upper-and-lower scale trimming strategy for the wavelet domain estimation from a practical standpoint. The other objective of this paper is to give a useful reference to the applied researchers and practitioners.

Keywords: finite sample performance comparison, Fourier frequency, GDP, non-stationary long memory time series, wavelet

1 Introduction

In this paper we consider a fractionally integrated I(d) process $\{X_t\}$ with spectral density $f(\lambda)$ ($-\pi < \lambda < \pi$) that can be approximated by $|\lambda|^{-2d}$ up

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to a multiplicative for the frequency $\lambda \to 0$, where the integration order d could be any real number. Precisely speaking, the I(d) process $\{X_t\}$ has the form as follows:

$$(1-L)^d(X_t-\mu)=u_t, \quad t=1,\ldots,n, \tag{1}$$

where L is the lag operator and d is called memory parameter. Even more, a constraint is imposed on $\{u_t\}$ that the stationary process $\{u_t\}$ is a linear process as follows: for $t=0,1,\ldots,$

$$u_t = C(L)\varepsilon_t \stackrel{\text{def}}{=} \sum_{j=0}^{\infty} c_j\varepsilon_{t-j}, \ \sum_{j=0}^{\infty} j|c_j| < \infty, \ C(1) \neq 0,$$

where $\varepsilon_t \sim N(0,\sigma^2)$ and σ is a constant and $\mathrm{E}\varepsilon_t^4 < \infty$. Furthermore, when $d < \frac{1}{2}$, the process $\{X_t\}$ is said to possess long memory; and when $d > 0$, $\{X_t\}$ is called stationary. As a special case of I(d) process, the stationary fractionally integrated autoregressive and moving average (ARFIMA) process (2), has been explored to a large extent in the literature, i.e. d belongs to the interval $[-\frac{1}{2},\frac{1}{2})$. However, in the real world, macroeconomic and financial data usually exhibit long memory but non-stationarity, therefore, in this paper our emphasis is laid on the estimation of $d > \frac{1}{2}$.

When the stationary process $\{X_t\}$ is generalized to the non-stationary case, especially when $d > 1$, the main obstacle encountered in the Fourier domain is that the most widely used periodogram will no longer properly approximate the true spectral density of the process. Three important manipulations to solve this problem could be found in the existing literature: (i) Differencing before estimation, but it incurs problems if the series is trend stationary ([17], [18]); (ii) Tapering ([20])/Tapering after differencing ([5]); and (iii) Utilization of the exact representation (or the representation derived from it) of the true spectral density proposed in [13]. We will implement the latter two manipulations of the data in our Monte Carlo experiment. In the wavelet domain scale spectrum is utilized instead of the periodogram in the estimation procedure ([10]), which will be detailed later. In this paper nine estimation procedures, for the non-stationary I(d) process, are recapped and they encompass parametric and semi-parametric Fourier/wavelet methods.

From a practical standpoint, there are at least two reasons to motivate the finite sample performance (FSP) comparison of the estimators mentioned in this paper. First, the the amount of macroeconomic data available to the applied researchers or practitioners is usually very limited. For example, the historical observations of GDP is less than 200 due to the quarterly release of the statistical institutes since or after 1970, so the estimator's FSP for the small sample size needs to be illustrated. Second, in [2], albeit for the stationary case, there exist obvious discrepancies between the memory parameter estimator $\hat{d}$'s for the exchange rates of six Asia Pacific countries, due to the different estimation procedures utilized; furthermore, even the same estimation procedure with different selections of frequency ordinates could result in very different memory estimators. Therefore, this phenomenon requires that we asses the adequacy of the estimation procedures utilizing various numbers of frequency ordinates in order to build up a solid background for their application to the real data. For these reasons, we carry out a Monte Carlo experiment for ARFIMA(p, d, q) process with $d \in [\frac{1}{2}, 2]$, the interval of d we encounter most in the real economic and financial world. Previously, [12] conducted a thorough investigation of the memory parameter estimators through a Monte Carlo experiment for the stationary time series, while in this paper we deal with the non-stationary I(d) process with $d > \frac{1}{2}$, concisely describe the widely used or recently developed estimation procedures applicable to this type of process and compare their FSP in terms of the bias and root mean square errors (RMSE). Finally, according to our simulation results, the relatively optimal estimation procedure will be applied to the macroeconomic data of 16 OECD countries in Europe.

The remainder of this paper is structured as follows. The ARFIMA(p, d, q) process is defined and explored in Section 2. The methodology and estimation techniques utilized in our analysis are concisely described in Section 3. The simulation setup and the comparison of the finite sample performances are reported in Section 4. The macroeconomic data descriptions and empirical findings are contained in Section **??**. The paper is concluded in Section 5. The notations of the mathematical symbols appearing in this paper could be found in Section 6. The tables and figures of FSP comparison are attached in the Appendix.

2 Methodology and Estimation Techniques

2.1 ARFIMA(p, d, q) Process

In this paper, we assess the estimation procedures through the simulation study for the ARFIMA(p, d, q) process covering both stationary and non-stationary cases, which has the form as follows:

$$\Phi(L)(1 - L)^d(X_t - \mu) = \Theta(L)\varepsilon_t, \quad d \in [0, 2], \tag{2}$$

where μ is a constant and $\varepsilon_t \sim N(0, \sigma^2)$. The polynomials $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$, $\Theta(L) = 1 - \theta_1 L - \ldots - \theta_q L^q$ do not share common roots and their roots are all strictly outside unit circle. We call d the memory parameter. The spectral density of the ARFIMA(p, d, q) process (2) is

$$f(\lambda) = \frac{\sigma^2}{2\pi}|1 - e^{i\lambda}|^{-2d}\frac{|\Theta(e^{i\lambda})|}{|\Phi(e^{i\lambda})|} = \frac{\sigma^2}{2\pi}(2\sin\frac{\lambda}{2})^{-2d}\frac{|\Theta(e^{i\lambda})|}{|\Phi(e^{i\lambda})|}. \quad (3)$$

Hence $f(\lambda)$ is even and infinitely differentiable. Let $f^*(\lambda) = \frac{|\Theta(e^{i\lambda})|}{|\Phi(e^{i\lambda})|}$ and due to its continuity, $f^*(\lambda)$ can be approximated by a constant in the small shrinking vicinity of the origin, that is,

$$f(\lambda) \sim G|1 - e^{i\lambda}| \sim G|\lambda|^{-2d}, \quad 0 < G < \infty, \text{ as } \quad \lambda \to 0, \quad (4)$$

hence we have, as $\lambda \to 0$

$$\log(f(\lambda)) \sim \log G + (-2\log|1 - e^{i\lambda}|) \cdot d \overset{\text{def}}{=} \log G + d \cdot g(\lambda), \quad 0 < G < \infty. \quad (5)$$

3 Semi-parametric Estimators

There exist two prevailing types of semi-parametric estimation approaches in the existing literature: one is in the Fourier domain and the other is in the wavelet domain. In this section we provide an overview of these methods.

The semi-parametric Fourier estimation procedure is based on the equation (3), therefore, the method is free from the specification of short-run dynamics in the considered process (2) and only the m frequency ordinates in a small neighborhood of the origin are involved in the estimation where the selection of the bandwidth parameter m is up to the applied researchers or practitioners.

The estimation procedures presented below are all general-purpose methods, which signify that the interval to which the memory parameter d belongs only needs to be loosely known.

3.1 Semi-parametric Estimators in Fourier Domain

When the I(d) process is generalized to cover the non-stationary ($d > \frac{1}{2}$) or non-invertible ($d < -\frac{1}{2}$) case, the usual periodogram is no longer a good approximation for the spectral density of $\{X_t\}$. To address this problem, several modifications to the discrete Fourier transformations (DFT), hence the periodogram were proposed. Therefore to begin with we introduce four types of DFT.

- **Hurvich and Chen (HC) Tapered DFT and its (Pooled) Periodogram**

The paper [5] considered a family of complex-valued taper to deal with non-invertible time series, a result of over-differencing the process. Let

$$h_t = 1 - e^{i\lambda_j},$$

and define the HC tapered DFT of order τ as follows:

$$\omega^t_{x,\tau}(\lambda_j) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi \sum h_t^2}} \sum_{t=1}^{n} h_t^{\tau} X_t e^{it\lambda_j} \tag{6}$$

where $\lambda_j = \frac{2\pi j}{n}$, $j = 1, 2, \ldots, n$ is the fundamental Fourier frequency. Then the tapered periodogram is

$$I^t_{x,\tau}(\lambda_j) = |\omega^t_{x,\tau}(\lambda_j)|^2. \tag{7}$$

When $h_t = 1,\ t = 1, \ldots, n$, then (6) and (7)become the usual DFT and periodogram, denoted as $\omega_x(\lambda_j)$ and $I_x(\lambda_j)$, respectively.Moreover, the pooled periodogram is defined as in [?], which is $I^{tp}_{x,\tau,p}(\tilde{\lambda}_k) = \sum_{j=(p+\tau)(k-1)+1}^{(p+\tau)(k-1)+p} I^t_{x,\tau}(\lambda_j)$, where $\tilde{\lambda}_k \stackrel{\text{def}}{=} p^{-1} \sum_{j=(p+\tau)(k-1)+1}^{(p+\tau)k} \lambda_j$.

- **Exact form of DFT (eDFT) and its Periodogram**
  The paper [13] presented an exact form of DFT which is resulted from a pure algebraic manipulation and hence applies to all values of d. The exact form of DFT (eDFT) is defined by

$$\omega^e_x(\lambda_j; d) \stackrel{\text{def}}{=} \omega_x(\lambda_j) - D_n(e^{i\lambda_j}; d)^{-1} \frac{1}{\sqrt{2\pi n}} \tilde{X}_{\lambda_j n}(d), \tag{8}$$

  and the exact form of periodogram then is $I^e_{\Delta^d X}(\lambda) = |D_n(e^{i\lambda_j}; d)|^2 |\omega^e_x(\lambda_j; d)|^2$, where $D_n(e^{i\lambda_j}; d) = \sum_{k=0}^{n} \frac{(-d)_k}{k!} e^{ik\lambda}$ and

$$\tilde{X}_{\lambda n}(d) = \sum_{p=0}^{n-1} \tilde{d}_{\lambda p} e^{-ip\lambda} X_{n-p} \quad \text{with } \tilde{d}_{\lambda p} = \sum_{k=p+1}^{n} \frac{(-d)_k}{k!} e^{ik\lambda}.$$

- **Modified DFT (modDFT) and its Periodogram**
  Inspired by (8), [14] conceived an approximation to (8) and called it the modified DFT (modDFT). This modDFT is invariant to a constant and a linear time trend, which is an advantage over the eDFT (8). The modDFT is defined by

$$\omega^{mod}_x(\lambda_j) \stackrel{\text{def}}{=} \omega_x(\lambda_j) + \frac{e^{i\lambda_j}}{1 - e^{i\lambda_j}} \frac{X_n - X_0}{\sqrt{2\pi n}}, \tag{9}$$

  and the modified periodogram is then $I^{mod}_x(\lambda_j) = |\omega^{mod}_x(\lambda_j)|^2$.

- **Fully-Extended DFT (fextDFT) and its Periodogram**
  Still suggested in the work of [13] for the I(d) process with $d \in \mathbb{R}$, the fextDFT was proposed in [1] and the resulting fully-extended periodogram can properly approximate the spectral density function in the small neighborhood of the origin. The fully-extended DFT (fextDFT) is defined on a compact set $I := [a, b] \subset [-\frac{3}{2}, \infty)$ by

$$\omega_x^{fext}(\lambda_j; d) \stackrel{\text{def}}{=} \omega_x(\lambda_j) + k(\lambda_j; d), \tag{10}$$

  where $k(\lambda_j; d)$ is the correction term taking constant values on the intervals $d \in I_p := [p - \frac{1}{2}, p + \frac{1}{2}),\ p = -1, 0, 1, 2, \ldots$, which is defined by

$$k(\lambda_j; d) := \begin{cases} -e^{i\lambda_j} Z_0, & d \in I_{-1} = [-3/2, -1/2); \\ 0, & d \in I_0 = [-1/2, 1/2); \\ e^{i\lambda_j} \sum_{r=1}^{p} (1 - e^{i\lambda_j})^{-r} Z_r, & d \in I_p,\ p = 1, 2, \ldots. \end{cases} \tag{11}$$

  where $Z_0 := \omega_x(0) = (2\pi n)^{-1/2} \sum_{t=1}^{n} X_t$ and $Z_r := (2\pi n)^{-1/2}(\nabla^{r-1} X_n - \nabla^{r-1} X_0)$, $r = 1, \ldots, p$.

  The fully-extended periodogram then is $I_x^{fext}(\lambda_j; d) = |\omega_x^{fext}(\lambda_j; d)|^2$.

3.1.1 Log-Periodogram Methods in Fourier Domain

The semi-parametric log-periodogram regression estimator envisioned by [4], often referred to as GPH estimator, is widely used in the empirical study to estimate the memory parameter d of the invertible and stationary I(d) process. Therefore when it comes to the non-stationary I(d) process with $d > \frac{1}{2}$, it is natural to stationarize the process through differencing before the application of the GPH estimation procedure. However, this manipulation sometimes results in an over-differenced I(d) process, i.e., $d < -\frac{1}{2}$, and hence cause the frequency leakage of the process, which motivates [5] to utilize the tapering to solve this problem.

We now introduce the estimation procedure proposed in [5]. Assume that δ^{th}-order difference for the I(d) process $\{X_t\}$, i.e., $Y_t \stackrel{\text{def}}{=} \Delta^\delta X_t$, leads to the stationary process $\{Y_t\}$, which means $d < \delta + \frac{1}{2}$ and the density function $f_y(\lambda)$ of $\{Y_t\}$ can be approximated by $C|\lambda|^{d-\delta}$ due to the relationship (4). Define $d_\delta = d - \delta$ here and after.

The tapered and pooled GPH estimator has the same spirit as the GPH estimator in that they are based on the linear regression model

$$\log(I_{y,\tau,p}^t) = \text{constant} - d_\delta g(\tilde{\lambda}_k) + \text{error}. \tag{12}$$

The tapered and pooled GPH (tpGPHF) estimator $\hat{d}_\delta^{tpGHP}$ is then the OLS estimator for the regression (12) and [7] showed that when $-\tau - 1/2 < d_\delta < 1/2$, for the properly selected bandwidth parameter m, we have

$$\sqrt{m}(\hat{d}_\delta^{tpGPH} - d_\delta) \to N(0, \frac{\sigma_{p,\tau}^2}{4}), \tag{13}$$

where $\sigma_{p,\tau}^2$ is decreasing with p. The tpGPHF estimator is then $\hat{d}^{tpGPH} = \delta + \hat{d}_\delta^{tpGPH}$.

3.1.2 Local Whittle (LWF) Methods in Fourier Domain

Besides the GPHF estimaton procedure, another popular method in the Fourier domain is the LWF estimation procedure suggested by [6] and investigated by [15] for the stationary time series, from which are derived the LWF estimators valid to the non-stationary process we consider in this subsection. In the following, we briefly review these LWF estimators, respectively.

- **The Tapered Local Whittle (tLWF) Estimator**
  For the tapered local Whittle (tLWF) estimator, the negative tapered LW objective function is

$$Q_{y,\tau}^{tLWF}(C, d_\delta) = \frac{1}{m}\sum_{j=1}^{m}\left[\log(C|1-e^{i\lambda_j}|^{-2d_\delta}) + \frac{I_{y,\tau}^t(\lambda_j)}{C|1-e^{i\lambda_j}|^{-2d_\delta}}\right]. \tag{14}$$

  then tLWF estimator $\hat{d}$ is obtained by $\hat{d}^{tLWF} = \hat{d}_\delta^{tLWF} + \delta$ where $(\hat{C}, \hat{d}_\delta^{tLWF})$ is the minimizer of (14). [7] showed that as $n \to \infty$ for the properly selected bandwidth parameter m, we have

$$\sqrt{m}(\hat{d}_\delta^{tLWF} - d_0) \to N(0, \frac{\Psi(\tau)}{4}),$$

  where $\Psi(\tau) = \frac{\Gamma(4\tau+1)\Gamma^4(\tau+1)}{\Gamma^4(2\tau+1)}$ and $\Gamma(\cdot)$ is the Gamma function.

  We now turn to the procedures that are all rooted in the work of [13].

- **The Exact Local Whittle (eLWF) Estimator**
  For the eLWF estimation procedure proposed in [17], the negative eLW objective function is given by

$$Q^{eLWF}(G, d) = \frac{1}{m}\sum_{j=1}^{m}\left[\log(G\lambda_j^{-2d}) + \frac{1}{G}I_{\Delta^d X}^e(\lambda_j)\right]. \tag{15}$$

The estimator $\hat{d}^{eLWF}$ is obtained by minimizing (15), that is,

$$(\hat{G}, \hat{d}^{eLWF}) = \text{argmin}_{G \in (0,\infty),\ d \in [\Delta_1, \Delta_2]} Q^{eLWF}(G, d),$$

where the length of $[\Delta_1, \Delta_2]$ is no bigger than $-\frac{9}{2}$ and $\Delta_1 \geq -1$. [17] showed that when $d \in (\Delta_1, \Delta_2)$ with the properly selected m, as $n \to \infty$, we have

$$\sqrt{m}(\hat{d}^{eLWF} - d_0) \to N(0, \frac{1}{4}).$$

In fact, for the empirical utilization the restriction imposed on the length of the interval is unnecessary. The drawback of this estimator is that it only applies to the zero-mean process. To overcome this problem, [16] proposed a feasible exact local Whittle estimator with detrending first (dtr-feLWF). The idea is to estimate the mean or detrend the process making use of the combination of $\bar{X}$ and X_1 before the application of the eLWF estimation procedure, where $\bar{X}$ is the average of the observation values $\{X_t\}_{t=1,\ldots,n}$. Then the feLWF estimator is consistent for $d > -\frac{1}{2}$ and when $d \in (-\frac{1}{2}, 2)$ with the properly selected m, we have

$$\sqrt{m}(\hat{d}^{feLWF} - d_0) \to N(0, \frac{1}{2}).$$

- **The Modified Local Whittle (modLWF) Estimator**
The modLWF estimator $\hat{d}^{modLWF}$ is the second ordinate of pair $(\hat{G}, \hat{d}^{modLWF})$ which minimizes of the negative modLW objective function defined as

$$Q^{modLWF}(G, d) = \frac{1}{m} \sum_{j=1}^{m} \left[\log(G\lambda_j^{-2d}) + \frac{I_x^{mod}(\lambda_j)}{G\lambda_j^{-2d}} \right] \tag{16}$$

[14] showed that the modLWF estimator $\hat{d}^{modLWF}$ is consistent for $d \in (0, 2)$; and for $d \in (\frac{1}{2}, \frac{7}{4})$ with the properly selected of m, as $n \to \infty$, we have

$$\sqrt{m}(\hat{d}^{modLWF} - d_0) \to N(0, \frac{1}{4}).$$

In his proof, X_0 is known, however, if the initial value is suitably selected, for instance, $X_0 = \mathcal{O}_P(1)$, then its value will not influence the large sample behavior of the estimator; see [14]. Moreover, we can also deduce from (9) that the modified DFT is invariant to a constant and/or a linear time trend hence so is the modLWF estimator.

- **The Fully-Extended Local Whittle (fextLWF) Estimator**
The fextLWF estimator was proposed in [1], which applies to the I(d) process with $d \in (-\frac{3}{2}, \infty)$. The negative fextLW objective function is

$$Q^{fextLWF}(G,d) = \frac{1}{m}\sum_{j=1}^{m}\left[\log(G\lambda_j^{-2d}) + \frac{I_x^{fext}(\lambda_j)}{G\lambda_j^{-2d}}\right],$$

and the fextLWF estimator $\hat{d}^{fextLWF}$ satisfies

$$(\hat{G}, \hat{d}^{fextLWF}) = \text{argmin}_{d\in(-3/2,\infty)} Q^{fextLWF}(G,d).$$

[1] showed that for the properly selected m, as $n \to \infty$, we have

$$\sqrt{m}(\hat{d}^{fext} - d_0) \to N(0, \frac{1}{4}).$$

Additionally, under the ARFIMA(p, d, q) setting, this estimator even works for the process $\{X_t\}$ with a polynomial time trend of degree p. Since the correction term (11) involves $X_t,\ t \leq 0$, the enumeration of the first p^{th} observations, $X_{1-p},\ X_{2-p},...,X_0$ is very crucial to the estimation results. However, we should bear it in mind that for the DFT $\omega_x(\lambda_j)$ in (10), the $\{X_t\}$ is always numbered as $X_1,\ X_2, \ldots, X_n$.

The fully-extended local polynomial Whittle (fextLPWF) estimation procedure proposed in [11] is a bias-reduced method derived from fextLWF method. It tweaks the fextLWF estimator in that the short-memory component of the spectrum is modeled by a finite, positive, even polynomial $\varphi(\lambda) = \sum_{l=1}^{r}\theta_l\lambda^{2r},\ r \in \mathbb{N}$ instead of the constant 'G' in equation (3), that is, the spectral density $f(\lambda)$ $(-\pi < \lambda < \pi)$ is approximated by

$$f(\lambda) \sim \varphi(\lambda)\lambda^{-2d}, \quad \lambda \to 0. \tag{17}$$

The even feature of φ is due to the symmetricity of the spectrum around 0. Compared to the constant case, it is supposed to gain an obvious bias reduction but at the expense of an increase in its RMSE. We implement the fextLPWF estimator with $\varphi(\lambda)$ being constant (corresponding to the fextLWF estimator) and a polynomial of order $r = 1$ in the Monte Carlo experiment.

Before concluding this section, we have two more comments to make. First, we implement the modLWF and feLWF/dtr-feLWF estimation procedures to estimate the memory parameter d of the "Type I" fractional process in this simulation study instead of "Type II" one which is the process the modLWF and feLWF/dtr-feLWF estimators designed for so that in this paper is manifested the empirical effectiveness of these estimation procedures for the former type process. Second, we are still left with a question of the selection of bandwidth parameter m in the estimation. Although there is no general optimal selection of m, we should remember that as $n \to \infty$, we have $m \to \infty$ and $\lim_{n\to\infty} \frac{m}{n} = 0$. We will illustrate in Section 4 how these estimators respond to m through the Monte Carlo experiment.

3.2 Wavelet Estimators

The other class of semi-parametric estimators considered in this paper is in the wavelet domain. Before presenting the wavelet estimation procedures, we need to first introduce the following assumptions as in [3] about the scale function $\phi \in \mathbf{L}^2(\mathbb{R})$ and the wavelet $\psi \in \mathbf{L}^2(\mathbb{R})$: Let

$$\hat{\phi}(\xi) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} \phi(t)e^{-i\xi t}dt \quad \text{and} \quad \hat{\psi}(\xi) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} \psi(t)e^{-i\xi t}dt$$

as the Fourier transforms of ϕ and ψ, respectively.

W-1 ϕ and ψ are compactly-supported, integrable, and $\hat{\phi}(0) = \int_{-\infty}^{\infty} \phi(t)dt = 1$ and $\int_{-\infty}^{\infty} \psi^2(t)dt = 1$.

W-2 There exists $\alpha > 1$ such that $\sup_{\xi\in\mathbb{R}} |\hat{\psi}|(1 + |\xi|)^{\alpha} < \infty$.

W-3 The function ψ has M vanishing moments, i.e. $\int_{-\infty}^{\infty} t^m\psi(t)dt = 0$ for all $m = 0, \ldots, M-1$.

W-4 The function $\sum_{k\in\mathbb{Z}} k^m\phi(\cdot - k)$ is a polynomial of degree m for all $m = 0, \ldots, M-1$.

In the wavelet semi-parametric context, for the process $\{X_t\}$ with a spectral density $f(\lambda) = |1 - e^{i\lambda}|^{-2d} f^*(\lambda)$, the wavelet estimation procedure utilizes the scale spectrum $\sigma_j^2(d, f^*)$ instead of the periodogram in the Fourier domain, that is, when $j \to \infty$,

$$\sigma_j^2(d, f^*) \stackrel{\text{def}}{=} var[W_{j,0}^X] \asymp \sigma^2 2^{2dj}, \tag{18}$$

where $\sigma_j^2(d, f^*)$ can be estimated by the empirical variance (see [9])

$$\hat{\sigma}_j^2 \stackrel{\text{def}}{=} \frac{1}{n_j} \sum_{k=0}^{n_j-1} (W_{j,k}^X)^2 \quad \text{for} \quad (j,k) \in l_n, \tag{19}$$

where $l_n \stackrel{\text{def}}{=} \{(j,k) : j \geq 0, 0 \leq k < n_j\}$ with

$$n_j = \lfloor 2^{-j}(n - T + 1) - T + 1 \rfloor \tag{20}$$

where $T = 2M$. Furthermore, define U the highest scale such that $U = \max\{j : \lfloor 2^{-j}(n-T+1)-T+1\rfloor \geq 0\}$ as well as L the lowest scale such that $L = 0$. Clearly, for the wavelet estimation, we are obliged to select the lower and upper scales, i.e., L_n and U_n, which satisfies $L \leq L_n < U_n \leq U$; then we have $m^{equiv} = \frac{1}{2}\sum_{j=L_n}^{U_n} n_j$, the half the number of wavelet coefficients involved in the wavelet estimations and it is a counterpart of bandwidth parameter m for the Fourier estimation procedure.

Remark 3.1 [9] showed that given $M \geq \delta$, the utilization of a wavelet and a scaling function satisfying (**W-4**) and (**W-3**) implicitly performed a M^{th}-order differentiation of the time series, which is contrast to the Fourier methods, where the time series must be explicitly differentiated at least δ times and a data taper must be applied on the differenced series to avoid frequency-domain leakage; see [5].

3.2.1 The Local Regression Wavelet (LRW) Estimation

Up to our knowledge, the existing literature only documents the asymptotic convergence for the first and second order moments of the LRW estimator in a Gaussian process context (which is a rare situation in the reality). Due to (18) and (19), we have the approximate regression relationship

$$\log(\hat{\sigma}_j^2) = \log \sigma^2 + dj(2\log 2) + \text{error}. \tag{21}$$

The LRW estimator is defined as the OLS estimator for (21) utilizing the scales from L_n to U_n and we will investigate the impact of selection L_n and U_n on the estimation performance through the Monte Carlo experiment. Under suitable conditions, for the Gaussian process $\{X_t\}$ with the suitably selected lower scale L_n, we have

$$\sqrt{n2^{-L_n}}(\hat{d}^{LRW} - d_0) \to N(0, \sum_{i,j=0}^{l} \mathbf{w}_j V_{i,j}(d)\mathbf{w}_j), \tag{22}$$

see [8].

3.2.2 The Local Whittle Wavelet (LWW) Estimation

The local Whittle wavelet likelihood function is given by

$$L^{LWW}(\sigma^2, d) = \frac{1}{2\sigma^2} \sum_{(j,k)\in l} 2^{-2dj}(W_{j,k}^X)^2 + \frac{|l|}{2} \log(\sigma^2 2^{2\langle l \rangle} d), \tag{23}$$

where l is l_n with $L_n \leq j \leq U_n$ and $|l|$ denotes the cardinal of l and $\langle l \rangle \stackrel{\text{def}}{=} |l|^{-1} \sum_{(j,k)\in l} j$. The estimator $\hat{d}^{LWW}$ is obtained by minimizing (23). [10] showed that when $d \in (0, M + \frac{1}{2})$ with L_n satisfying

$$\lim_{n\to\infty} \{L_n^2 (n2^{-L_n})^{-1/4} + L_n^{-1}\} = 0,$$

we have

$$\hat{d}^{LWW} = d_0 + \mathcal{O}_{\mathbb{P}}\{(n2^{-L_n})^{-1/2} + 2^{-\beta L_n}\}. \tag{24}$$

Hence, for $2^{L_n} \asymp n^{1/(12\beta)}$, the optimal rate $n^{\beta/(1+2\beta)}$ is obtained, which theoretically supports the idea of truncating the lowest wavelet scale indices when the LWW estimation procedure is implemented. Furthermore, [10] also showed the central limit theorem (CLT) for the LWW estimator of the process $\{X_t\}$, but unfortunately, $\{X_t\}$ is confined to a Gaussian process.

Remark 3.2 The simulation study in [10] indicates that, for the LWW estimators, it makes no big differences among different wavelet filters.

3.3 Parametric Estimators

As a comparison estimation procedure to the semi-parametric methods, we now briefly introduce two parametric methods: one is in time domain and the other is in Fourier domain.

3.3.1 Maximum Likelihood in the Time Domain (MLT)

Under the model specification (2), the maximum likelihood estimator ($\hat{d}^{MLT}$, $\hat{\phi}_1, \ldots, \hat{\phi}_p, \hat{\theta}_1, \ldots, \hat{\theta}_q$) in the time domain is the maximizer of the log-likelihood defined as

$$L^{MLT}(d, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q) = -\frac{n}{2} \log \left[\sum_{j=1}^{n} (\Phi(L)\Theta^{-1}(L)(1-L)^d X_j)^2 \right].$$

[19] established the $\sqrt{n}$-consistency an asymptotic normality for $(\hat{d}^{MLT}, \hat{\phi}_1, \ldots, \hat{\phi}_p, \hat{\theta}_1, \ldots, \hat{\theta}_q)$; see p.564 and p.568 in [19]. In this paper, we do not implement this procedure in our simulation study since it is too computationally demanded and it needs the exact specification of the mechanism by which the data is generated.

4 Finite Sample Comparison

4.1 Monte Carlo Setup

The ARFIMA(p, d, q) processes for $d \in [\frac{1}{2}, 2]$ are simulated using Matlab with the ToolboxLRD toolbox; see [3]. The bias and RMSE were computed with 5,000 replications. The sample size n and number of frequency involved in the estimation m are selected to be $n = 150, 300, 500, 1000$ and $m = n^{\alpha}$ with $\alpha = 0.5$, 0.65 as well as 0.8, respectively. The numeric results in the Tables 1-56 were obtained by rounding the Matlab outputs to four decimal places.

We execute the Monte Carlo experiments for the following three models within the ARFIMA(p, d, q) settings:

$$(1 - L)^d (X_t - \mu) = u_t, \tag{25}$$

where

$$(a)\ u_t = \varepsilon_t; \tag{26}$$
$$(b)\ (1 - \phi L) u_t = \varepsilon_t; \tag{27}$$
$$(c)\ u_t = (1 + \theta L) \varepsilon_t, \tag{28}$$

where $\varepsilon_t \sim N(0, 1)$; and we call ϕ the autoregressive (AR) coefficient and θ the moving average (MA) coefficient.

4.2 Monte Carlo Results for Fourier Estimators

For all the semi-parametric Fourier estimators we consider in this paper, the consistency is revealed in the Tables 1-42, since the bias and RMSE becomes smaller as the sample size increases. From the Tables 1-42, for short-run dynamics contaminated time series, except for fextLPWF ($r = 1$) estimator, we observe that when the bandwidth parameter m moves from $\lfloor n^{.5} \rfloor$ to $\lfloor n^{.65} \rfloor$, the RMSE of the estimators is usually reduced at the expense of an inflation in the bias, so there is a trade-off between the estimator's performance of the bias and the RMSE. However, still except for fextLPWF with $r = 1$, when m moves to $\lfloor n^{.8} \rfloor$ we would have an increment in the bias and RMSE hence the very big m is not suitable for the estimation. For the GPHF and tapered LWF estimation procedures, it requires $\tau \geq \delta$ (see [5]) and for simplicity, we set $\delta = \tau$ throughout this Monte Carlo experiment and we find that the estimator obtained from 1^{st}-order over-differenced data exhibits the bigger RMSE than that of the stationarized one since the RMSE expectedly climbs along with the growth of the taper order; see the Tables 5-21. In the following, we describe the FSP of these Fourier estimators respectively.

- **GPHF Estimator**
  For the moderate or large sample size, the pooling expectedly helps reduce the RMSE, but the no-pooled GPHF estimator usually enjoys a little smaller bias, which is revealed in the Tables 3-12. However, when it comes to the GPHF estimator of the small sample size, i.e., 150 in this experiment, contrary to the theoretical result, the pooling of order 3 inflates the RMSE for $d \in [\frac{3}{2}, 2]$; and although the pooling yields the smaller RMSE for $d \in [\frac{1}{2}, \frac{3}{2})$, the resulting estimator is subject to a big increase in bias. Therefore, it is not advised to pool the periodograms for the small sample size, while the pooled GPHF estimator is favored for the moderate or large sample size.

- **LWF Estimator**
  Four types of local Whittle estimators are discussed in this part, respectively. First, concerning the tapered LW estimation procedure, among the simulation results listed in the Tables 16-20, under the ARFIMA$(0, d, 0)$ model specification, with respect to $m = \lfloor n^{.5} \rfloor$ and $\lfloor n^{.65} \rfloor$, we find that when m is raised up to $\lfloor n^{.8} \rfloor$ the bias and RMSE generally decrease. For the process with the error term contaminated by the ARMA process, the tLWF estimator obtained from the stationarized data with $m = \lfloor n^{.65} \rfloor$ enjoys the smallest RMSE, while the estimator obtained from the stationarized or 1^{st}-order over-differenced data with $m = \lfloor n^{.5} \rfloor$ usually has the smallest bias. Furthermore, we observe that the moderate negative MA coefficient worsens the FSP of the estimator the most among all the process simulated in this experiment.

  Second, concerning the fextLPWF estimation procedure, we implement it with $r = 0$ (corresponding to the fextLWF estimator) and $r = 1$ in this experiment. Although the consistency and asymptotic normality of the fextLPWF estimator is not proved for $d = \frac{1}{2}$ and $\frac{3}{2}$, we observe that the fextLPWF estimation procedure still practically works for these points; see the first four columns of the Tables 25-38. For the fextLPWF estimator with $r = 1$, when the bandwidth parameter m is raised from $\lfloor n^{.5} \rfloor$ to $\lfloor n^{.65} \rfloor$, it gains the bias and RMSE reduction. This results from the fact that the fextLPWF estimation procedure imposes a lower-bound requirement on m in order to achieve the consistency and asymptotic normality of $(\hat{d}^{fextLPWF},\ \hat{\theta})$, that is, $\frac{m^{2r+1/2}}{n^{2r}} \to \infty$; and for instance, if m is chosen to be n^{α}, $\alpha > 0$ and $r = 1$, we get $\alpha > .8$. From the Tables 25-38, we find that when the error term contaminated by the ARMA process with positive AR or MA coefficient, the fextLPWF estimator with $m = \lfloor n^{.8} \rfloor$ expectedly outperforms the estimator with $m = \lfloor n^{.65} \rfloor$; nonetheless, if the ARMA process has the negative AR or MA coefficient, then there is a trade-off between the performance of the bias and RMSE when m is raised from $m = \lfloor n^{.65} \rfloor$ to $m = \lfloor n^{.8} \rfloor$. Therefore, from a practical standpoint, it is favored to choose a relatively bigger bandwidth parameter compared to the other LWF estimators, but α is not necessarily bigger than .8. The Figures 1-3 illustrate the sensitivity of the fextLPWF estimator $\hat{d}^{fextLPWF}$ to the bandwidth parameter m and obviously, the fextLPWF estimator is inclined to bigger bandwidth m with respect to other LWF estimators; see the Figures 4-8. From the Tables 34-42, see the results for the fextLWF estimator with $m = \lfloor n^{.65}$ and the fextLPWF estimator with $m = \lfloor n^{.8} \rfloor$, clearly, the negative MA coefficient θ worsens the FSP of the estimator much more that the positive on with respect to the bias and RMSE; moreover, a big positive θ (=.8), impairs the bias performance much while it leads to a similar RMSE performance compared to the case with a moderate θ, i.e., .4. Compared to the fextLWF estimator with $m = \lfloor n^{.5} \rfloor$, the fextLPWF suffers a bad performance in terms of the bias and RMSE; with $m = \lfloor n^{.65} \rfloor$, the fextLPWF estimator has a big advantage of the bias while the fextLWF estimator enjoys a much smaller RMSE. Additionally, for the empirical utilization, the selection of the fextLWF with $m = \lfloor n^{.5} \rfloor / \lfloor n^{.65} \rfloor$ or the fextLPWF with $m = \lfloor n^{.65} \rfloor / \lfloor n^{.8} \rfloor$ is up to the applied researchers or the practitioners since these estimators have their own advantages. However, it needs to be pointed out that the fextLPWF estimator is much more computationally burdened than the fextLWF estimator.

Third, concerning the modLWF estimator, we observe a jump in bias at the points $d = \frac{1}{2}$ and $d = 2$ revealed in the Figures 9, therefore, it is not an ideal estimator for d near $d = \frac{1}{2}$ or 2 and the RMSE of this estimator is essentially the same except the small neighborhood of $d = 2$. Furthermore, under the ARFIMA$(0, d, 0)$ model specification, from the Tables 22-24, when m moves from $\lfloor n^{.5} \rfloor$ to $\lfloor n^{.65} \rfloor$, the RMSE consistently decreases as the bias performance varies according to the values of d; and when m moves up to $\lfloor n^{.8} \rfloor$, except for the d's in the small neighborhood around 2, i.e., $d = 1.8$, 1.9 and 2, the RMSE is generally reduced at the expense of a really big increment in bias. The following discussion is based on the estimation with $m = \lfloor n^{.65} \rfloor$. Under the ARFIMA$(1, d, 0)$ model specification, from the Tables 25-33, compared to the positive AR coefficient ϕ, the negative one commonly worsens the FSP of estimator more in terms of the bias and RMSE; and with respect to the moderate ϕ, the big positive ϕ usually induces a small inflation in the bias and RMSE. Under the ARFIMA$(0, d, 1)$ model specification, from the Tables 34-42, as is the case for the fextLWF and fextLPWF estimators, the negative θ has a much more adverse impact on the bias and RMSE performance compared to the positive one; and the augmentation of the positive θ, i.e., .4 to .8 in this simulation study, generally inflates the bias and RMSE a little.

Finally, concerning the feLWF and dtr-feLWF estimation procedures, under the ARFIMA$(0, d, 0)$ model specification, if m moves from $\lfloor n^{.5} \rfloor$ to $\lfloor n^{.65} \rfloor$, the RMSE is reduced over the interval we consider and the bias may increase for several d's; and if $\lfloor n^{.8} \rfloor$ frequencies are utilized in the estimation instead of $\lfloor n^{.65} \rfloor$ frequencies, the reduction of RMSE is achieved with a big inflation in bias. For these two estimators utilizing $m(= \lfloor n^{.65} \rfloor)$ frequency ordinates, from the Tables 25-42, under the ARFIMA$(1, d, 0)$ or ARFIMA$(0, d, 1)$ model specification, we observe that the FSP of the feLWF/dtr-feLWF estimator is in line with the situation of the modLWF estimator. Furthermore, we make a comparison between them utilizing with $\lfloor n^{.5} \rfloor$ and $\lfloor n^{.65} \rfloor$. When the I(d) process is generated by the ARFIMA$(1, d, 0)$ with $\phi = .4$ and $\phi = .8$ as well as the ARFIMA$(0, d, 1)$ with $\theta = -.4$ and $\theta = .4$, for $d \in [.5, 1.1]$, the feLWF estimator usually outperforms the dtr-feLWF estimator in terms of the bias and RMSE while for $d \in (1.1, 2]$ the situation is just the opposite. As a particular case, when the I(d) process is generated by the ARFIMA$(1, d, 0)$ with negative AR coefficient, i.e., $\phi = -.4$ in this experiment, we observe that the feLWF estimator enjoys the lower RMSE compared to the dtr-feLWF estimator for $d \in [.5, 1.1]$ and the dtr-feLWF estimator has the lower RMSE for $d \in (1.1, 2]$, and they have the comparable bias performance in the whole interval we consider. Therefore, it is suggested that we utilize the feLWF estimator for the smaller part of the interval $[.5, 2]$ and otherwise the dtr-feLWF is recommended for the I(d) process with no linear time trend. Nonetheless, the feLWF estimator is not invariant to the linear time trend (see [16]), so we suggest the utilization of the dtr-feLWF estimator in empirical study.

4.3 Monte Carlo Results for Wavelet Estimators

In this Monte Carlo experiment, we provide the simulation results utilizing the Daubechies wavelets with vanishing moment $M = 2$. The reason behind this selection of M is that for small and moderate sample size, the bigger M is selected the smaller the number of wavelet coefficients is involved in the estimation; see (20) and hence the too big M will cause an inflation in the bias and RMSE. Through this simulation study we investigate the effects of the wavelet scales trimming on the performance of the LRW and LWW estimators and then provide some instructions on the trimming strategy. Taking a quick look at the Tables 44-55, we find that the consistency of the LRW and LWW estimators holds since the RMSE decreases as the sample size increases and the RMSE is generally smaller with more scales involved in the wavelet estimation; see the Figure 10 and 11. For both LRW and LWW estimators, we also find that the memory parameter estimator of d is very sensitive to the selection of the lower and upper scale indices utilized in the estimation procedures, especially for the small sample size and the trimming of the smallest lower-scales usually gains a big bias reduction, which results from the fact that lower-scale wavelet coefficients reflect the short/intermediate memory feature rather than the long memory feature of the process, and it is theoretically supported by (24). Furthermore, the RMSE of the wavelet estimator decreases as the value of d increases, which is probably because the implicit M^{th}-order differencing renders the I(d) process with small d over-differenced during the computation of the wavelet coefficient. Generally speaking, if the error term u_t in (28) takes on the moderate negative MA coefficient, this error term impairs the effectiveness of the LRW/LWW estimator considerably, especially for the small and moderate sample size; see the Table 47 and Table 54. Therefore, the LRW/LWW estimator is not suitable to the process whose error term is contaminated with by the ARMA process with big AR coefficients or negative MA coefficients. In the following, we turn to the FSP comparisons of the LRW and LWW estimator, respectively.

- **LRW Estimator**
  From the Tables 44-46, under the ARFIMA$(0, d, 0)$ model specification, the trimming of $K = 1$ and $J = 1$ leads to the estimators with the lowest RMSE while the estimators with the trimming of $K = 1$ and $J = 2$ enjoys the lowest bias. Under the ARFIMA$(1, d, 0)$ model specification, when AR coefficient ϕ takes on values $-.4$, $.4$ and $.8$, the bias and RMSE of the LRW estimator both inflate along with the growth of ϕ and it could be perceived that the AR coefficient has a big impact on the estimator's performance. Besides, for the short-run dynamics contaminated error term with positive AR or MA coefficient, we find that the trimming of the largest upper scale is conductive to the RMSE reduction of the LRW estimator and even the bias reduction in the positive MA coefficient case, which may be due to the very limited wavelet coefficients at the largest scale hence the spectral spectrum (19) could not be well approximated.

- **LWW Estimator**

Under the ARFIMA$(0, d, 0)$ model specification, the trimming strategy of $K = 0$ and $J = 1$ yields the estimators with the lowest RMSE, which empirically implies the fact that the more scales involved in the estimation the smaller the RMSE is. As is the case for the LRW estimator, from the Tables 52-53, the big AR coefficient has an obvious impact on the performance of the LWW estimator, that is, the bias and RMSE mount; and from the Tables 54, we find that the inclusion of the MA process with $\theta < 0$ into the error term also yields a not so ideal estimation result in terms of the bias and RMSE. Furthermore, not like the LRW estimator, for this LWW estimator, the trimming of the largest upper scales is not necessary at all.

Overall, among the models we consider in this experiment, for the error term in (28) with positive AR coefficient or in (28) with negative MA coefficient, the LWW estimator has a better estimation performance over the LRW estimator in terms of the bias and RMSE; and for the other cases, these two estimators have their own merits.

To sum up this subsection, we observe that the wavelet estimation procedures generally cannot compete with the Fourier ones in terms of the bias and RMSE. Therefore, for the empirical utilization, the Fourier domain estimation procedure is recommended and the relatively optimal estimator depends on the small interval to which d belongs as well as on the user's preference on the better performance of the bias or the RMSE.

5 Conclusion

In this paper, for non-stationary time series, we evaluate via bias and RMSE the adequacy of the estimation procedures presented above by conducting simulation experiment; and suggest two competitive estimation procedures, i.e., dtr-feLWF and fextLPWF estimators, for the practical utilization. As a by-product, we investigate the impacts of tapering and pooling on the performance of estimators for (12) and (14) in the frequency domain; and we propose suitable truncations of upper and lower scale indices for the wavelet estimation procedures in order to obtain relatively optimal estimators. Generally speaking, the wavelet estimators can not compete with the Fourier ones and through this simulation study we find that the fextLPWF estimator with $m = \lfloor n^{.8} \rfloor$ enjoys the lowest RMSE, therefore this estimator is favored for the practical utilization.

6 Notation

i.i.d process ε_t: i.e., $\varepsilon_t \sim N(0, \sigma^2)$, where σ is a constant;
$\sim$: ratio of the left and right hand sides tending to one in the limit
$\stackrel{\text{def}}{=}$: defined as;
L: lag operator, for instance, $LX_t = X_{t-1}$;
$\lambda_j = \frac{2\pi j}{n}$, $j = 1, \ldots, n$: fundamental Fourier frequency;
$\lfloor x \rfloor$: greatest integer less than x;
$f(x) = \mathcal{O}_P(g(x))$: There is a sufficiently large constant M such that for all sufficiently large values of x, $f(x)$ is at most M multiplied by $g(x)$ in absolute value in probability.
$\asymp$: asymptotically equal;
K: trimming number of the highest wavelet scales;
J: trimming number of the lowest wavelet scales;
The following are the abbreviations of the estimation methods.
feLWF estimation: feasible local Whittle Fourier estimation;
fextLPWF estimation: fully-extended local polynomial Whittle estimation;
GPHF estimation: tapered and pooled log-regression Fourier estimation;
MLT estimation: maximum likelihood estimation in time domain;
modLWF estimation: modified local Whittle Fourier estimation;
tLWF estimation: tapered local Whittle Fourier estimation;
LRW estimation: log-regression wavelet estimation;
LWW estimation: local Whittle wavelet estimation.

7 Appendix

7.1 Tables

The Monte Carlo simulation results are listed in the Tables 1-56.

Table 1: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 0)$ Model with $p = 1$

			$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	1	-0.0009	0.3897	0.0016	0.2337	0.0006	0.1518	2	0.0767	0.5003	0.0501	0.2997	0.0224	0.1871
	300	1	0.0028	0.3067	-0.0025	0.1698	0.0001	0.1055	2	0.0597	0.4359	0.0284	0.2166	0.0132	0.1355
	500	1	-0.0038	0.2478	-0.0021	0.1384	-0.0020	0.0842	2	0.0367	0.3270	0.0217	0.1796	0.0120	0.1047
	1000	1	-0.0034	0.2083	0.0034	0.1049	0.0002	0.0622	2	0.0347	0.2520	0.0177	0.1338	0.0058	0.0756
.7	150	1	0.0034	0.3842	-0.0048	0.2362	-0.0048	0.1529	2	0.0431	0.4967	0.0333	0.3009	0.0173	0.1885
	300	1	-0.0072	0.3123	-0.0022	0.1697	-0.0019	0.1060	2	0.0402	0.4089	0.0198	0.2181	0.0134	0.1314
	500	1	-0.0078	0.2512	0.0009	0.1389	-0.0013	0.0840	2	0.0301	0.3273	0.0163	0.1788	0.0076	0.1027
	1000	1	-0.0003	0.2005	-0.0015	0.1062	0.0005	0.0612	2	0.0273	0.2544	0.0086	0.1294	0.0059	0.0772
.9	150	1	-0.0076	0.3818	-0.0037	0.2362	-0.0040	0.1518	2	0.0394	0.5090	0.0251	0.3058	0.0138	0.1879
	300	1	0.0025	0.3038	-0.0016	0.1697	-0.0010	0.1074	2	0.0215	0.4167	0.0138	0.2135	0.0087	0.1313
	500	1	0.0023	0.2476	0.0015	0.1362	0.0006	0.0846	2	0.0273	0.3204	0.0151	0.1747	0.0041	0.1030
	1000	1	0.0052	0.2004	-0.0008	0.1060	0.0005	0.0622	2	0.0166	0.2551	0.0038	0.1337	0.0018	0.0746
1.1	150	1	-0.0056	0.3821	0.0076	0.2348	0.0030	0.1484	2	0.0223	0.4945	0.0152	0.3047	0.0079	0.1869
	300	1	0.0013	0.3056	0.0024	0.1702	-0.0014	0.1051	2	0.0229	0.4197	0.0096	0.2177	-0.0002	0.1346
	500	1	0.0027	0.2450	-0.0006	0.1380	-0.0009	0.0831	2	0.0181	0.3188	0.0066	0.1767	0.0022	0.1035
	1000	1	0.0021	0.2011	0.0025	0.1060	0.0012	0.0617	2	0.0088	0.2546	0.0021	0.1330	0.0026	0.0760
1.3	150	1	0.0126	0.3879	0.0107	0.2378	0.0039	0.1499	2	0.0170	0.4961	0.0085	0.2946	0.0027	0.1875
	300	1	0.0076	0.3051	0.0037	0.1676	0.0024	0.1082	2	0.0113	0.4166	0.0068	0.2206	0.0011	0.1319
	500	1	0.0091	0.2490	0.0025	0.1402	0.0006	0.0820	2	0.0071	0.3312	0.0050	0.1765	-0.0000	0.1030
	1000	1	0.0070	0.2018	0.0053	0.1086	0.0004	0.0613	2	0.0062	0.2556	0.0031	0.1363	0.0016	0.0758
1.5	150	2	-0.0130	0.5035	0.0025	0.2970	0.0009	0.1891	3	0.0744	0.6352	0.0454	0.3592	0.0234	0.2266
	300	2	0.0003	0.4127	-0.0013	0.2141	-0.0007	0.1332	3	0.0571	0.4847	0.0314	0.2529	0.0193	0.1541
	500	2	0.0035	0.3206	0.0010	0.1757	-0.0017	0.1037	3	0.0387	0.4029	0.0202	0.2039	0.0121	0.1213
	1000	2	-0.0009	0.2491	-0.0019	0.1342	-0.0004	0.0766	3	0.0412	0.3193	0.0193	0.1543	0.0082	0.0886
1.7	150	2	-0.0115	0.4963	0.0030	0.2975	-0.0092	0.1871	3	0.0378	0.6265	0.0290	0.3592	0.0190	0.2251
	300	2	0.0088	0.4109	-0.0086	0.2199	-0.0025	0.1319	3	0.0514	0.4894	0.0258	0.2512	0.0126	0.1577
	500	2	0.0041	0.3225	0.0008	0.1752	-0.0022	0.1036	3	0.0316	0.4083	0.0202	0.2082	0.0101	0.1190
	1000	2	0.0001	0.2525	-0.0003	0.1320	0.0006	0.0767	3	0.0338	0.3241	0.0140	0.1549	0.0069	0.0882
1.9	150	2	-0.0066	0.5024	-0.0014	0.3025	0.0004	0.1863	3	0.0410	0.6284	0.0203	0.3646	0.0064	0.2280
	300	2	-0.0002	0.4218	0.0032	0.2123	0.0014	0.1341	3	0.0325	0.4834	0.0134	0.2534	0.0099	0.1572
	500	2	0.0077	0.3217	0.0019	0.1739	0.0019	0.1033	3	0.0238	0.4065	0.0142	0.2025	0.0050	0.1194
	1000	2	-0.0053	0.2544	-0.0032	0.1327	-0.0003	0.0758	3	0.0165	0.3212	0.0060	0.1552	0.0047	0.0880

[a] The data generation is $(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 2: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 0)$ Model with $p = 3$

			$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	1	-0.0097	0.3511	-0.0010	0.2006	-0.0031	0.1245	2	0.0598	0.2625	0.0582	0.2617	0.0369	0.1659
	300	1	-0.0054	0.2733	0.0006	0.1396	-0.0034	0.0869	2	0.0460	0.1874	0.0401	0.1842	0.0227	0.1121
	500	1	-0.0079	0.2304	-0.0017	0.1153	0.0007	0.0667	2	0.0325	0.1489	0.0313	0.1495	0.0169	0.0860
	1000	1	0.0020	0.1789	-0.0007	0.0851	0.0000	0.0495	2	0.0241	0.1173	0.0215	0.1157	0.0109	0.0624
.7	150	1	-0.0091	0.3499	-0.0011	0.1979	-0.0041	0.1246	2	0.0458	0.2646	0.0400	0.2621	0.0259	0.1614
	300	1	-0.0148	0.2668	-0.0027	0.1401	-0.0003	0.0869	2	0.0338	0.1854	0.0203	0.1839	0.0180	0.1096
	500	1	-0.0071	0.2249	-0.0020	0.1126	-0.0021	0.0663	2	0.0243	0.1478	0.0240	0.1479	0.0112	0.0871
	1000	1	-0.0026	0.1779	-0.0018	0.0828	-0.0016	0.0493	2	0.0145	0.1108	0.0142	0.1114	0.0091	0.0619
.9	150	1	-0.0082	0.3432	0.0012	0.1965	-0.0033	0.1231	2	0.0292	0.2576	0.0257	0.2571	0.0156	0.1546
	300	1	-0.0061	0.2674	-0.0012	0.1379	-0.0013	0.0850	2	0.0178	0.1820	0.0205	0.1833	0.0124	0.1094
	500	1	-0.0097	0.2249	-0.0027	0.1122	-0.0015	0.0665	2	0.0173	0.1432	0.0122	0.1452	0.0066	0.0852
	1000	1	-0.0030	0.1723	-0.0012	0.0851	-0.0006	0.0482	2	0.0095	0.1110	0.0108	0.1106	0.0055	0.0614
1.1	150	1	0.0025	0.3470	0.0043	0.1970	0.0017	0.1253	2	0.0083	0.2504	0.0152	0.2492	0.0091	0.1507
	300	1	0.0056	0.2677	0.0054	0.1372	0.0030	0.0857	2	0.0117	0.1763	0.0075	0.1754	0.0040	0.1057
	500	1	0.0038	0.2231	0.0024	0.1093	0.0016	0.0647	2	0.0037	0.1451	0.0100	0.1429	0.0037	0.0832
	1000	1	0.0042	0.1751	0.0004	0.0852	0.0003	0.0485	2	0.0056	0.1098	0.0045	0.1107	0.0020	0.0599
1.3	150	1	0.0307	0.3389	0.0206	0.1950	0.0141	0.1252	2	-0.0020	0.2510	-0.0012	0.2474	0.0021	0.1532
	300	1	0.0259	0.2670	0.0146	0.1388	0.0100	0.0853	2	-0.0014	0.1802	0.0023	0.1749	0.0031	0.1083
	500	1	0.0200	0.2244	0.0110	0.1104	0.0065	0.0666	2	0.0004	0.1425	-0.0024	0.1436	0.0009	0.0845
	1000	1	0.0171	0.1750	0.0060	0.0844	0.0033	0.0488	2	0.0017	0.1090	0.0027	0.1095	-0.0007	0.0596
1.5	150	2	-0.0039	0.5700	-0.0022	0.2438	-0.0018	0.1500	3	0.0494	0.3245	0.0523	0.3228	0.0322	0.1893
	300	2	-0.0071	0.3685	0.0015	0.1771	0.0004	0.1056	3	0.0439	0.2364	0.0395	0.2339	0.0228	0.1318
	500	2	-0.0064	0.2925	-0.0029	0.1432	-0.0006	0.0832	3	0.0295	0.1752	0.0299	0.1760	0.0166	0.1004
	1000	2	-0.0033	0.2112	-0.0015	0.1083	-0.0013	0.0594	3	0.0232	0.1358	0.0217	0.1327	0.0122	0.0729
1.7	150	2	-0.0157	0.5661	-0.0033	0.2445	-0.0039	0.1490	3	0.0468	0.3174	0.0329	0.3159	0.0186	0.1877
	300	2	-0.0070	0.3615	0.0008	0.1736	-0.0012	0.1048	3	0.0277	0.2341	0.0298	0.2329	0.0175	0.1300
	500	2	-0.0133	0.2881	0.0005	0.1400	-0.0020	0.0836	3	0.0211	0.1727	0.0231	0.1745	0.0108	0.0993
	1000	2	-0.0031	0.2138	-0.0011	0.1058	-0.0024	0.0600	3	0.0160	0.1320	0.0138	0.1334	0.0064	0.0714
1.9	150	2	-0.0111	0.5581	-0.0055	0.2451	-0.0010	0.1530	3	0.0199	0.3174	0.0283	0.3125	0.0176	0.1839
	300	2	-0.0063	0.3659	-0.0011	0.1726	-0.0009	0.1041	3	0.0233	0.2296	0.0188	0.2322	0.0099	0.1307
	500	2	0.0035	0.2864	0.0017	0.1404	-0.0023	0.0827	3	0.0126	0.1712	0.0116	0.1763	0.0072	0.0973
	1000	2	0.0040	0.2062	-0.0036	0.1063	-0.0014	0.0584	3	0.0101	0.1314	0.0108	0.1296	0.0034	0.0717

[a] The data generation is $(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 3: Monte Carlo Results of the GPHF Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$ and $p = 1$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	1	0.0510	0.3773	0.1244	0.2683	0.2823	0.3196	2	0.1171	0.5159	0.1822	0.3506	0.3014	0.3536
		300	1	0.0207	0.3057	0.0885	0.1911	0.2327	0.2554	2	0.0673	0.4311	0.1197	0.2480	0.2478	0.2805
		500	1	0.0183	0.2495	0.0648	0.1531	0.2033	0.2193	2	0.0643	0.3271	0.0808	0.1933	0.2171	0.2410
		1000	1	0.0025	0.1997	0.0364	0.1124	0.1675	0.1784	2	0.0441	0.2518	0.0524	0.1426	0.1748	0.1902
	.7	150	1	0.0438	0.3781	0.1283	0.2695	0.2745	0.3126	2	0.0898	0.5112	0.1547	0.3384	0.2973	0.3507
		300	1	0.0158	0.3095	0.0889	0.1904	0.2281	0.2521	2	0.0561	0.4269	0.1069	0.2432	0.2392	0.2743
		500	1	0.0093	0.2472	0.0639	0.1539	0.2017	0.2181	2	0.0466	0.3234	0.0743	0.1952	0.2161	0.2392
		1000	1	0.0044	0.2006	0.0389	0.1145	0.1692	0.1801	2	0.0283	0.2587	0.0520	0.1432	0.1737	0.1897
	.9	150	1	0.0427	0.3841	0.1225	0.2652	0.2749	0.3131	2	0.0752	0.5020	0.1511	0.3321	0.2913	0.3469
		300	1	0.0167	0.3065	0.0886	0.1951	0.2298	0.2526	2	0.0478	0.4217	0.0990	0.2382	0.2368	0.2716
		500	1	0.0125	0.2465	0.0612	0.1517	0.2037	0.2202	2	0.0394	0.3257	0.0663	0.1875	0.2108	0.2345
		1000	1	0.0089	0.2007	0.0419	0.1136	0.1671	0.1781	2	0.0250	0.2597	0.0469	0.1403	0.1736	0.1888
	1.1	150	1	0.0558	0.3883	0.1288	0.2723	0.2753	0.3148	2	0.0660	0.5131	0.1432	0.3353	0.2852	0.3410
		300	1	0.0244	0.3141	0.0905	0.1912	0.2288	0.2527	2	0.0323	0.4155	0.0939	0.2356	0.2345	0.2685
		500	1	0.0124	0.2529	0.0644	0.1547	0.2039	0.2204	2	0.0190	0.3273	0.0655	0.1889	0.2109	0.2340
		1000	1	0.0150	0.1979	0.0400	0.1150	0.1689	0.1799	2	0.0099	0.2556	0.0428	0.1391	0.1688	0.1857
	1.3	150	1	0.0577	0.3826	0.1340	0.2715	0.2763	0.3153	2	0.0607	0.4995	0.1408	0.3249	0.2833	0.3408
		300	1	0.0367	0.3121	0.0930	0.1931	0.2296	0.2526	2	0.0315	0.4117	0.0868	0.2329	0.2347	0.2693
		500	1	0.0204	0.2487	0.0666	0.1554	0.2068	0.2230	2	0.0193	0.3246	0.0601	0.1857	0.2059	0.2309
		1000	1	0.0187	0.2006	0.0402	0.1106	0.1696	0.1808	2	0.0068	0.2584	0.0399	0.1383	0.1706	0.1860
	1.5	150	2	0.0567	0.5105	0.1216	0.3265	0.2804	0.3377	3	0.1265	0.6317	0.1722	0.3947	0.3069	0.3818
		300	2	0.0127	0.4235	0.0899	0.2318	0.2298	0.2650	3	0.0756	0.5004	0.1231	0.2805	0.2446	0.2891
		500	2	0.0202	0.3323	0.0628	0.1882	0.2040	0.2293	3	0.0670	0.4194	0.0861	0.2178	0.2209	0.2505
		1000	2	0.0189	0.2555	0.0413	0.1373	0.1699	0.1863	3	0.0420	0.3197	0.0560	0.1624	0.1775	0.1981
	1.7	150	2	0.0476	0.5067	0.1396	0.3251	0.2848	0.3407	3	0.1161	0.6309	0.1671	0.3970	0.2922	0.3690
		300	2	0.0133	0.4147	0.0788	0.2342	0.2260	0.2618	3	0.0585	0.4857	0.1058	0.2736	0.2440	0.2887
		500	2	0.0158	0.3278	0.0601	0.1887	0.2020	0.2266	3	0.0490	0.4028	0.0839	0.2149	0.2166	0.2472
		1000	2	0.0027	0.2598	0.0363	0.1386	0.1697	0.1855	3	0.0308	0.3187	0.0548	0.1642	0.1743	0.1956
	1.9	150	2	0.0452	0.5053	0.1321	0.3202	0.2778	0.3353	3	0.0747	0.6257	0.1541	0.3854	0.2932	0.3684
		300	2	0.0210	0.4156	0.0870	0.2308	0.2268	0.2626	3	0.0455	0.4845	0.1094	0.2767	0.2372	0.2845
		500	2	0.0040	0.3264	0.0564	0.1878	0.2064	0.2307	3	0.0391	0.4079	0.0737	0.2180	0.2108	0.2427
		1000	2	0.0059	0.2504	0.0364	0.1369	0.1683	0.1850	3	0.0252	0.3165	0.0498	0.1620	0.1742	0.1956

[a] The data generation is $(1 + .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d - \delta \in [-.5, .5)/d - \delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d - \delta < .5/-1.5 \leq d - \delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 4: Monte Carlo Results of the GPHF Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$ and $p = 3$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	1	0.0535	0.3614	0.1368	0.2449	0.2790	0.3046	2	0.1771	0.6083	0.2090	0.3372	0.3323	0.3681
		300	1	0.0180	0.2666	0.0892	0.1667	0.2304	0.2461	2	0.0922	0.4016	0.1398	0.2314	0.2654	0.2864
		500	1	0.0169	0.2281	0.0619	0.1300	0.2063	0.2166	2	0.0844	0.3163	0.0969	0.1770	0.2198	0.2364
		1000	1	0.0057	0.1783	0.0374	0.0948	0.1684	0.1755	2	0.0504	0.2306	0.0622	0.1275	0.1833	0.1930
	.7	150	1	0.0454	0.3486	0.1305	0.2388	0.2797	0.3062	2	0.1328	0.5943	0.1986	0.3281	0.3248	0.3607
		300	1	0.0186	0.2711	0.0862	0.1651	0.2302	0.2453	2	0.0811	0.3977	0.1243	0.2214	0.2585	0.2804
		500	1	0.0037	0.2205	0.0633	0.1288	0.2047	0.2151	2	0.0574	0.3064	0.0865	0.1685	0.2181	0.2340
		1000	1	0.0025	0.1760	0.0369	0.0927	0.1699	0.1766	2	0.0364	0.2251	0.0568	0.1253	0.1797	0.1898
	.9	150	1	0.0496	0.3463	0.1348	0.2381	0.2789	0.3048	2	0.0976	0.5820	0.1770	0.3098	0.3131	0.3505
		300	1	0.0202	0.2673	0.0903	0.1643	0.2327	0.2475	2	0.0541	0.3741	0.1127	0.2133	0.2526	0.2746
		500	1	0.0131	0.2254	0.0659	0.1299	0.2061	0.2165	2	0.0456	0.3012	0.0778	0.1640	0.2112	0.2279
		1000	1	0.0046	0.1747	0.0371	0.0938	0.1685	0.1753	2	0.0258	0.2222	0.0482	0.1216	0.1788	0.1889
	1.1	150	1	0.0597	0.3468	0.1446	0.2426	0.2829	0.3096	2	0.0661	0.5775	0.1616	0.2985	0.3031	0.3402
		300	1	0.0270	0.2667	0.0933	0.1675	0.2334	0.2490	2	0.0376	0.3828	0.1059	0.2064	0.2469	0.2685
		500	1	0.0213	0.2212	0.0669	0.1288	0.2081	0.2180	2	0.0275	0.2930	0.0732	0.1625	0.2075	0.2238
		1000	1	0.0086	0.1718	0.0410	0.0941	0.1701	0.1770	2	0.0193	0.2130	0.0447	0.1181	0.1743	0.1846
	1.3	150	1	0.0917	0.3523	0.1672	0.2590	0.2950	0.3194	2	0.0688	0.5810	0.1573	0.2948	0.3008	0.3370
		300	1	0.0507	0.2684	0.1108	0.1768	0.2404	0.2553	2	0.0225	0.3705	0.1027	0.2053	0.2419	0.2651
		500	1	0.0377	0.2245	0.0746	0.1347	0.2133	0.2231	2	0.0152	0.2962	0.0670	0.1602	0.1724	0.1829
		1000	1	0.0222	0.1737	0.0470	0.0962	0.1731	0.1800	2	0.0010	0.2122	0.0412	0.1176	0.2050	0.2210
	1.5	150	2	0.0326	0.5642	0.1477	0.2898	0.3001	0.3355	3	0.1571	0.6300	0.2057	0.3752	0.3353	0.3828
		300	2	0.0186	0.3659	0.0926	0.1974	0.2401	0.2623	3	0.1106	0.6182	0.1293	0.2657	0.2585	0.2904
		500	2	0.0176	0.2893	0.0625	0.1536	0.2050	0.2206	3	0.0780	0.4090	0.0982	0.2023	0.2287	0.2489
		1000	2	0.0062	0.2128	0.0373	0.1138	0.1721	0.1822	3	0.0518	0.2655	0.0620	0.1450	0.1813	0.1949
	1.7	150	2	0.0465	0.5580	0.1520	0.2904	0.2955	0.3321	3	0.1194	0.6046	0.1955	0.3748	0.3264	0.3752
		300	2	0.0127	0.3736	0.0908	0.1949	0.2371	0.2594	3	0.0749	0.6067	0.1149	0.2543	0.2514	0.2842
		500	2	0.0116	0.2904	0.0631	0.1545	0.2038	0.2203	3	0.0681	0.4062	0.0852	0.1950	0.2240	0.2442
		1000	2	-0.0003	0.2121	0.0363	0.1133	0.1721	0.1818	3	0.0449	0.2630	0.0547	0.1421	0.1795	0.1927
	1.9	150	2	0.0428	0.5577	0.1485	0.2889	0.2982	0.3347	3	0.1139	0.6013	0.1720	0.3582	0.3131	0.3634
		300	2	0.0250	0.3644	0.0950	0.1972	0.2404	0.2623	3	0.0498	0.5903	0.1048	0.2481	0.2449	0.2768
		500	2	0.0173	0.2892	0.0617	0.1539	0.2050	0.2208	3	0.0409	0.3953	0.0783	0.1895	0.2194	0.2399
		1000	2	-0.0006	0.2080	0.0396	0.1137	0.1707	0.1804	3	0.0226	0.2593	0.0480	0.1411	0.1753	0.1886

[a] The data generation is $(1+.4L)(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 5: Monte Carlo Results of the GPHF Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .4$ and $p = 1$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	1	-0.0098	0.3742	-0.0361	0.2375	-0.1566	0.2191	2	0.0444	0.5128	0.0058	0.3020	-0.1354	0.2307
		300	1	-0.0051	0.3048	-0.0200	0.1694	-0.1094	0.1542	2	0.0563	0.4262	0.0068	0.2163	-0.0901	0.1611
		500	1	0.0021	0.2488	-0.0111	0.1396	-0.0875	0.1214	2	0.0358	0.3325	0.0155	0.1754	-0.0791	0.1307
		1000	1	-0.0045	0.1996	-0.0075	0.1051	-0.0626	0.0873	2	0.0368	0.2583	0.0081	0.1339	-0.0552	0.0939
	.7	150	1	-0.0173	0.3763	-0.0436	0.2453	-0.1548	0.2171	2	0.0282	0.5148	-0.0083	0.3096	-0.1430	0.2379
		300	1	-0.0096	0.3090	-0.0243	0.1716	-0.1060	0.1512	2	0.0396	0.4232	0.0011	0.2168	-0.0961	0.1637
		500	1	-0.0069	0.2471	-0.0159	0.1390	-0.0890	0.1209	2	0.0292	0.3340	0.0037	0.1767	-0.0790	0.1303
		1000	1	-0.0026	0.2004	-0.0088	0.1070	-0.0634	0.0879	2	0.0264	0.2544	0.0026	0.1315	-0.0589	0.0969
	.9	150	1	-0.0177	0.3837	-0.0290	0.2456	-0.1556	0.2173	2	0.0133	0.5099	-0.0135	0.3027	-0.1464	0.2383
		300	1	-0.0089	0.3062	-0.0153	0.1700	-0.1076	0.1516	2	0.0270	0.4167	-0.0094	0.2115	-0.1031	0.1699
		500	1	-0.0037	0.2463	-0.0121	0.1391	-0.0884	0.1214	2	0.0163	0.3247	-0.0015	0.1764	-0.0831	0.1333
		1000	1	0.0018	0.2006	-0.0086	0.1084	-0.0621	0.0870	2	0.0111	0.2534	0.0009	0.1320	-0.0593	0.0975
	1.1	150	1	-0.0056	0.3849	-0.0351	0.2413	-0.1493	0.2111	2	-0.0026	0.5038	-0.0205	0.3018	-0.1535	0.2422
		300	1	-0.0012	0.3127	-0.0200	0.1709	-0.1092	0.1527	2	0.0131	0.4177	-0.0111	0.2153	-0.1039	0.1669
		500	1	-0.0039	0.2527	-0.0119	0.1371	-0.0845	0.1186	2	0.0117	0.3225	-0.0086	0.1778	-0.0864	0.1326
		1000	1	0.0079	0.1975	-0.0077	0.1069	-0.0606	0.0859	2	0.0124	0.2566	-0.0051	0.1327	-0.0610	0.0965
	1.3	150	1	-0.0031	0.3771	-0.0251	0.2368	-0.1503	0.2111	2	0.0001	0.4900	-0.0362	0.2989	-0.1575	0.2437
		300	1	0.0112	0.3104	-0.0114	0.1709	-0.1056	0.1503	2	0.0063	0.4132	-0.0194	0.2209	-0.1054	0.1703
		500	1	0.0044	0.2477	-0.0073	0.1386	-0.0833	0.1178	2	-0.0057	0.3272	-0.0137	0.1789	-0.0854	0.1344
		1000	1	0.0115	0.2005	-0.0055	0.1063	-0.0622	0.0868	2	0.0048	0.2520	-0.0063	0.1331	-0.0620	0.0979
	1.5	150	2	-0.0092	0.5091	-0.0314	0.2949	-0.1622	0.2486	3	0.0516	0.6302	0.0040	0.3631	-0.1266	0.2530
		300	2	-0.0118	0.4233	-0.0235	0.2159	-0.1037	0.1691	3	0.0535	0.4823	0.0061	0.2496	-0.0900	0.1810
		500	2	0.0046	0.3315	-0.0108	0.1776	-0.0869	0.1354	3	0.0396	0.4032	0.0034	0.2040	-0.0771	0.1444
		1000	2	0.0115	0.2551	-0.0085	0.1326	-0.0637	0.1000	3	0.0332	0.3161	0.0109	0.1565	-0.0551	0.1052
	1.7	150	2	-0.0175	0.5043	-0.0367	0.3049	-0.1589	0.2451	3	0.0394	0.6269	-0.0011	0.3613	-0.1353	0.2617
		300	2	-0.0117	0.4148	-0.0232	0.2232	-0.1072	0.1740	3	0.0339	0.4921	0.0009	0.2589	-0.0935	0.1806
		500	2	0.0001	0.3273	-0.0137	0.1773	-0.0881	0.1364	3	0.0371	0.3966	0.0035	0.1987	-0.0808	0.1450
		1000	2	-0.0046	0.2596	-0.0082	0.1365	-0.0626	0.0983	3	0.0312	0.3184	0.0028	0.1549	-0.0549	0.1036
	1.9	150	2	-0.0193	0.5037	-0.0353	0.2973	-0.1626	0.2497	3	0.0177	0.6361	-0.0189	0.3607	-0.1412	0.2607
		300	2	-0.0036	0.4148	-0.0201	0.2176	-0.1100	0.1710	3	0.0093	0.4875	-0.0057	0.2458	-0.0988	0.1851
		500	2	-0.0115	0.3264	-0.0169	0.1812	-0.0880	0.1365	3	0.0195	0.4023	-0.0040	0.2051	-0.0832	0.1449
		1000	2	-0.0015	0.2505	-0.0089	0.1331	-0.0626	0.0995	3	0.0261	0.3089	0.0021	0.1547	-0.0588	0.1059

[a] The data generation is $(1-.4L)(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 6: Monte Carlo Results of the GPHF Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .4$ and $p = 3$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	1	-0.0170	0.3580	-0.0456	0.2061	-0.1536	0.1992	2	0.0743	0.6031	0.0140	0.2606	-0.1363	0.2094
		300	1	-0.0110	0.2663	-0.0231	0.1439	-0.1077	0.1374	2	0.0776	0.4008	0.0181	0.1859	-0.0901	0.1425
		500	1	0.0016	0.2275	-0.0168	0.1130	-0.0914	0.1135	2	0.0588	0.3096	0.0183	0.1502	-0.0677	0.1100
		1000	1	-0.0011	0.1783	-0.0076	0.0872	-0.0633	0.0802	2	0.0505	0.2283	0.0166	0.1142	-0.0510	0.0787
	.7	150	1	-0.0253	0.3469	-0.0420	0.2042	-0.1523	0.1954	2	0.0517	0.5827	-0.0039	0.2523	-0.1508	0.2184
		300	1	-0.0105	0.2708	-0.0283	0.1436	-0.1108	0.1416	2	0.0387	0.3897	0.0040	0.1794	-0.0988	0.1472
		500	1	-0.0116	0.2208	-0.0198	0.1126	-0.0903	0.1123	2	0.0460	0.3039	0.0097	0.1455	-0.0752	0.1141
		1000	1	-0.0043	0.1761	-0.0107	0.0856	-0.0641	0.0809	2	0.0302	0.2194	0.0094	0.1138	-0.0579	0.0848
	.9	150	1	-0.0212	0.3435	-0.0411	0.2028	-0.1582	0.2016	2	0.0216	0.5902	-0.0231	0.2570	-0.1581	0.2210
		300	1	-0.0089	0.2668	-0.0234	0.1397	-0.1070	0.1373	2	0.0238	0.3853	-0.0064	0.1783	-0.1070	0.1497
		500	1	-0.0022	0.2251	-0.0118	0.1146	-0.0891	0.1107	2	0.0267	0.2967	-0.0050	0.1459	-0.0814	0.1180
		1000	1	-0.0022	0.1747	-0.0090	0.0848	-0.0637	0.0804	2	0.0209	0.2178	0.0012	0.1114	-0.0586	0.0843
	1.1	150	1	-0.0107	0.3418	-0.0334	0.1976	-0.1497	0.1933	2	0.0122	0.5693	-0.0315	0.2541	-0.1648	0.2250
		300	1	-0.0022	0.2652	-0.0211	0.1396	-0.1034	0.1354	2	0.0222	0.3791	-0.0134	0.1762	-0.1112	0.1538
		500	1	0.0060	0.2203	-0.0118	0.1141	-0.0876	0.1093	2	0.0086	0.2951	-0.0045	0.1410	-0.0823	0.1176
		1000	1	0.0018	0.1716	-0.0091	0.0863	-0.0624	0.0784	2	0.0051	0.2135	-0.0037	0.1101	-0.0620	0.0863
	1.3	150	1	0.0207	0.3403	-0.0152	0.1988	-0.1402	0.1866	2	-0.0039	0.5606	-0.0442	0.2530	-0.1762	0.2313
		300	1	0.0216	0.2643	-0.0044	0.1370	-0.1004	0.1317	2	0.0020	0.3829	-0.0169	0.1750	-0.1163	0.1583
		500	1	0.0224	0.2224	-0.0060	0.1106	-0.0824	0.1050	2	-0.0020	0.2919	-0.0108	0.1422	-0.0857	0.1206
		1000	1	0.0154	0.1729	-0.0025	0.0853	-0.0586	0.0762	2	0.0003	0.2146	-0.0038	0.1069	-0.0643	0.0874
	1.5	150	2	-0.0276	0.5645	-0.0396	0.2466	-0.1745	0.2317	3	0.0788	0.6112	0.0190	0.3202	-0.1383	0.2334
		300	2	-0.0095	0.3657	-0.0260	0.1769	-0.1178	0.1578	3	0.0659	0.6146	0.0218	0.2348	-0.0840	0.1552
		500	2	0.0014	0.2887	-0.0163	0.1436	-0.0876	0.1215	3	0.0686	0.4046	0.0186	0.1763	-0.0752	0.1232
		1000	2	-0.0018	0.2127	-0.0105	0.1081	-0.0648	0.0875	3	0.0434	0.2668	0.0121	0.1335	-0.0508	0.0894
	1.7	150	2	-0.0136	0.5564	-0.0480	0.2513	-0.1783	0.2326	3	0.0576	0.6089	0.0054	0.3118	-0.1517	0.2412
		300	2	-0.0154	0.3735	-0.0243	0.1761	-0.1203	0.1585	3	0.0309	0.6091	0.0090	0.2301	-0.0913	0.1597
		500	2	-0.0045	0.2902	-0.0158	0.1422	-0.0862	0.1187	3	0.0371	0.3972	0.0101	0.1738	-0.0780	0.1230
		1000	2	-0.0082	0.2122	-0.0110	0.1076	-0.0638	0.0868	3	0.0289	0.2681	0.0098	0.1323	-0.0551	0.0900
	1.9	150	2	-0.0177	0.5563	-0.0412	0.2407	-0.1749	0.2279	3	0.0190	0.6085	-0.0210	0.3155	-0.1574	0.2444
		300	2	-0.0031	0.3636	-0.0224	0.1751	-0.1162	0.1567	3	0.0252	0.6080	-0.0006	0.2288	-0.0993	0.1624
		500	2	0.0013	0.2886	-0.0173	0.1393	-0.0869	0.1198	3	0.0232	0.3944	-0.0003	0.1687	-0.0849	0.1280
		1000	2	-0.0086	0.2082	-0.0099	0.1076	-0.0651	0.0883	3	0.0212	0.2559	0.0031	0.1298	-0.0584	0.0916

[a] The data generation is $(1 - .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 7: Monte Carlo Results of the GPHF Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .8$ and $p = 1$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	1	-0.0022	0.3878	-0.0361	0.2380	-0.2131	0.2596	2	0.1354	0.4971	-0.0080	0.2926	-0.1957	0.2710
		300	1	-0.0023	0.3122	-0.0228	0.1735	-0.1430	0.1777	2	0.0551	0.4349	0.0036	0.2180	-0.1271	0.1839
		500	1	-0.0003	0.2433	-0.0175	0.1432	-0.1118	0.1391	2	0.0335	0.3266	0.0030	0.1730	-0.1033	0.1451
		1000	1	0.0009	0.2038	-0.0080	0.1071	-0.0794	0.1007	2	0.0333	0.2520	0.0030	0.1340	-0.0725	0.1051
	.7	150	1	-0.0157	0.3791	-0.0435	0.2407	-0.2155	0.2639	2	0.0287	0.4965	-0.0141	0.2973	-0.2049	0.2778
		300	1	-0.0124	0.3118	-0.0262	0.1691	-0.1423	0.1797	2	0.0354	0.4089	-0.0030	0.2154	-0.1322	0.1878
		500	1	-0.0015	0.2473	-0.0142	0.1385	-0.1138	0.1411	2	0.0271	0.3270	-0.0025	0.1770	-0.1061	0.1471
		1000	1	-0.0017	0.2021	-0.0098	0.1075	-0.0807	0.1019	2	0.0260	0.2542	0.0031	0.1344	-0.0730	0.1055
	.9	150	1	-0.0158	0.3744	-0.0417	0.2403	-0.2147	0.2622	2	0.0262	0.5072	-0.0306	0.3026	-0.2072	0.2786
		300	1	0.0020	0.3068	-0.0310	0.1752	-0.1445	0.1794	2	0.0168	0.4167	-0.0100	0.2153	-0.1373	0.1924
		500	1	-0.0061	0.2462	-0.0195	0.1388	-0.1131	0.1399	2	0.0243	0.3202	-0.0050	0.1752	-0.1102	0.1512
		1000	1	-0.0032	0.2023	-0.0086	0.1080	-0.0804	0.1017	2	0.0152	0.2549	-0.0019	0.1340	-0.0742	0.1061
	1.1	150	1	-0.0072	0.3787	-0.0376	0.2378	-0.2107	0.2604	2	0.0090	0.4924	-0.0310	0.3082	-0.2148	0.2837
		300	1	-0.0068	0.3065	-0.0258	0.1728	-0.1401	0.1753	2	0.0180	0.4196	-0.0272	0.2179	-0.1385	0.1926
		500	1	-0.0072	0.2502	-0.0152	0.1410	-0.1122	0.1400	2	0.0151	0.3182	-0.0085	0.1751	-0.1153	0.1566
		1000	1	0.0038	0.2015	-0.0093	0.1083	-0.0801	0.1019	2	0.0074	0.2546	-0.0064	0.1331	-0.0785	0.1100
	1.3	150	1	0.0010	0.3814	-0.0324	0.2386	-0.2094	0.2571	2	0.0039	0.4941	-0.0467	0.3050	-0.2199	0.2866
		300	1	-0.0012	0.3095	-0.0202	0.1710	-0.1404	0.1769	2	0.0063	0.4166	-0.0237	0.2223	-0.1418	0.1950
		500	1	0.0052	0.2514	-0.0161	0.1398	-0.1132	0.1400	2	0.0041	0.3306	-0.0142	0.1795	-0.1154	0.1559
		1000	1	0.0004	0.2072	-0.0041	0.1072	-0.0765	0.0978	2	0.0048	0.2557	-0.0070	0.1318	-0.0785	0.1091
	1.5	150	2	-0.0046	0.5014	-0.0451	0.3039	-0.2216	0.2890	3	0.0598	0.6328	0.0012	0.3507	-0.1866	0.2918
		300	2	0.0063	0.4308	-0.0232	0.2151	-0.1407	0.1933	3	0.0514	0.4836	0.0082	0.2552	-0.1236	0.1992
		500	2	-0.0048	0.3264	-0.0185	0.1767	-0.1138	0.1547	3	0.0358	0.4024	0.0054	0.2051	-0.1032	0.1573
		1000	2	-0.0037	0.2538	-0.0119	0.1340	-0.0799	0.1108	3	0.0400	0.3192	0.0067	0.1561	-0.0692	0.1120
	1.7	150	2	-0.0107	0.5027	-0.0525	0.3072	-0.2193	0.2869	3	0.0230	0.6255	-0.0164	0.3594	-0.1916	0.2967
		300	2	0.0053	0.4074	-0.0232	0.2152	-0.1433	0.1950	3	0.0455	0.4883	-0.0089	0.2535	-0.1331	0.2056
		500	2	-0.0070	0.3332	-0.0134	0.1756	-0.1143	0.1534	3	0.0287	0.4080	0.0019	0.2027	-0.1068	0.1595
		1000	2	-0.0019	0.2540	-0.0103	0.1321	-0.0797	0.1098	3	0.0325	0.3241	-0.0006	0.1554	-0.0752	0.1164
	1.9	150	2	-0.0094	0.5099	-0.0500	0.3094	-0.2240	0.2910	3	0.0267	0.6284	-0.0263	0.3597	-0.1963	0.2990
		300	2	-0.0054	0.4180	-0.0300	0.2166	-0.1407	0.1941	3	0.0266	0.4833	-0.0167	0.2518	-0.1332	0.2040
		500	2	-0.0017	0.3284	-0.0191	0.1754	-0.1131	0.1519	3	0.0209	0.4065	-0.0046	0.2006	-0.1083	0.1616
		1000	2	0.0008	0.2514	-0.0104	0.1354	-0.0825	0.1128	3	0.0152	0.3211	-0.0053	0.1543	-0.0751	0.1163

[a] The data generation is $(1-.8L)(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 8: Monte Carlo Results of the GPHF Estimation for ARFIMA$(1,d,0)$ Model with $\phi=.8$ and $p=3$

				$d-\delta\in[-.5,.5)$							$d-\delta\in[-1.5,-.5)$					
				$m=\lfloor n^{.5}\rfloor$		$m=\lfloor n^{.65}\rfloor$		$m=\lfloor n^{.8}\rfloor$			$m=\lfloor n^{.5}\rfloor$		$m=\lfloor n^{.65}\rfloor$		$m=\lfloor n^{.8}\rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	1	-0.0230	0.3489	-0.0474	0.2041	-0.2087	0.2433	2	0.0958	0.5997	0.0080	0.2558	-0.2097	0.2604
		300	1	-0.0097	0.2709	-0.0298	0.1464	-0.1425	0.1674	2	0.0741	0.3926	0.0176	0.1825	-0.1292	0.1688
		500	1	-0.0110	0.2279	-0.0187	0.1134	-0.1164	0.1342	2	0.0577	0.3081	0.0147	0.1461	-0.0924	0.1265
		1000	1	-0.0075	0.1786	-0.0082	0.0852	-0.0797	0.0934	2	0.0451	0.2303	0.0144	0.1157	-0.0702	0.0930
	.7	150	1	-0.0254	0.3497	-0.0524	0.2045	-0.2112	0.2449	2	0.0429	0.5901	-0.0063	0.2607	-0.2204	0.2703
		300	1	-0.0123	0.2682	-0.0295	0.1428	-0.1443	0.1683	2	0.0324	0.3921	0.0054	0.1824	-0.1392	0.1763
		500	1	-0.0116	0.2240	-0.0232	0.1153	-0.1182	0.1357	2	0.0457	0.3019	0.0065	0.1459	-0.0994	0.1310
		1000	1	-0.0055	0.1738	-0.0119	0.0854	-0.0802	0.0939	2	0.0302	0.2221	0.0047	0.1099	-0.0738	0.0954
	.9	150	1	-0.0157	0.3448	-0.0498	0.1991	-0.2127	0.2468	2	0.0394	0.5867	-0.0228	0.2569	-0.2318	0.2790
		300	1	-0.0064	0.2639	-0.0301	0.1412	-0.1432	0.1678	2	0.0346	0.3839	-0.0105	0.1814	-0.1438	0.1798
		500	1	-0.0059	0.2228	-0.0208	0.1151	-0.1156	0.1330	2	0.0191	0.2968	-0.0005	0.1422	-0.1033	0.1339
		1000	1	-0.0057	0.1786	-0.0101	0.0851	-0.0800	0.0934	2	0.0173	0.2226	-0.0002	0.1106	-0.0763	0.0975
	1.1	150	1	0.0023	0.3421	-0.0461	0.2004	-0.2031	0.2379	2	0.0133	0.5791	-0.0437	0.2541	-0.2354	0.2799
		300	1	0.0016	0.2657	-0.0256	0.1383	-0.1392	0.1636	2	0.0086	0.3790	-0.0167	0.1767	-0.1489	0.1828
		500	1	0.0015	0.2236	-0.0121	0.1133	-0.1151	0.1323	2	0.0157	0.2981	-0.0141	0.1457	-0.1071	0.1363
		1000	1	0.0022	0.1726	-0.0075	0.0851	-0.0772	0.0912	2	0.0085	0.2141	-0.0042	0.1098	-0.0797	0.1004
	1.3	150	1	0.0221	0.3397	-0.0257	0.1973	-0.1973	0.2328	2	-0.0159	0.5637	-0.0541	0.2568	-0.2453	0.2882
		300	1	0.0200	0.2629	-0.0100	0.1411	-0.1314	0.1579	2	0.0033	0.3726	-0.0298	0.1827	-0.1496	0.1821
		500	1	0.0209	0.2227	-0.0051	0.1123	-0.1105	0.1285	2	-0.0095	0.2929	-0.0174	0.1436	-0.1098	0.1383
		1000	1	0.0168	0.1734	-0.0016	0.0837	-0.0756	0.0895	2	0.0022	0.2119	-0.0081	0.1093	-0.0802	0.0998
	1.5	150	1	-0.0144	0.5688	-0.0501	0.2488	-0.2424	0.2848	2	0.0613	0.6097	-0.0013	0.3208	-0.2030	0.2745
		300	1	-0.0069	0.3707	-0.0319	0.1777	-0.1541	0.1864	2	0.0787	0.6131	0.0196	0.2331	-0.1148	0.1753
		500	1	-0.0047	0.2909	-0.0225	0.1427	-0.1114	0.1386	2	0.0595	0.4032	0.0119	0.1731	-0.1024	0.1410
		1000	1	-0.0078	0.2119	-0.0105	0.1085	-0.0819	0.1016	2	0.0445	0.2697	0.0135	0.1344	-0.0691	0.1003
	1.7	150	1	-0.0253	0.5581	-0.0613	0.2502	-0.2437	0.2872	2	0.0297	0.5945	-0.0040	0.3141	-0.2155	0.2853
		300	1	-0.0187	0.3681	-0.0328	0.1777	-0.1554	0.1882	2	0.0562	0.5985	0.0033	0.2325	-0.1210	0.1767
		500	1	-0.0122	0.2890	-0.0215	0.1420	-0.1102	0.1383	2	0.0548	0.4010	0.0035	0.1714	-0.1061	0.1458
		1000	1	-0.0058	0.2116	-0.0129	0.1084	-0.0832	0.1023	2	0.0344	0.2654	0.0063	0.1312	-0.0714	0.1011
	1.9	150	1	-0.0206	0.5572	-0.0533	0.2486	-0.2456	0.2878	2	0.0188	0.6059	-0.0309	0.3184	-0.2286	0.2932
		300	1	-0.0141	0.3607	-0.0293	0.1736	-0.1539	0.1855	2	0.0439	0.6115	-0.0012	0.2285	-0.1306	0.1828
		500	1	-0.0041	0.2856	-0.0183	0.1411	-0.1118	0.1389	2	0.0269	0.3949	-0.0051	0.1707	-0.1100	0.1447
		1000	1	-0.0039	0.2092	-0.0098	0.1070	-0.0823	0.1017	2	0.0205	0.2611	0.0004	0.1310	-0.0748	0.1028

[a] The data generation is $(1-.8L)(1-L)^dX_t=\varepsilon_t$, $\varepsilon_t\sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta\in[-.5,.5)/d-\delta\in[-1.5,-.5)$ is such that for the true value d, select a δ satisfying $-.5\le d-\delta<.5/-1.5\le d-\delta<-.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 9: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$ and $p = 1$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	1	-0.0424	0.3874	-0.1255	0.2703	-0.2733	0.3131	2	0.0104	0.4992	-0.0925	0.3135	-0.2541	0.3158
		300	1	-0.0199	0.3074	-0.0883	0.1909	-0.2263	0.2502	2	0.0240	0.4233	-0.0576	0.2264	-0.2143	0.2523
		500	1	-0.0121	0.2512	-0.0598	0.1492	-0.2032	0.2195	2	0.0343	0.3227	-0.0350	0.1769	-0.1936	0.2197
		1000	1	-0.0074	0.2009	-0.0397	0.1146	-0.1678	0.1787	2	0.0270	0.2616	-0.0197	0.1340	-0.1647	0.1816
	.7	150	1	-0.0541	0.3901	-0.1238	0.2695	-0.2758	0.3134	2	-0.0092	0.4942	-0.1013	0.3199	-0.2664	0.3248
		300	1	-0.0206	0.3149	-0.0918	0.1938	-0.2291	0.2524	2	0.0228	0.4161	-0.0609	0.2236	-0.2197	0.2572
		500	1	-0.0156	0.2479	-0.0615	0.1502	-0.2014	0.2182	2	0.0103	0.3319	-0.0432	0.1790	-0.1977	0.2230
		1000	1	-0.0133	0.1986	-0.0388	0.1152	-0.1680	0.1789	2	0.0193	0.2575	-0.0277	0.1363	-0.1639	0.1807
	.9	150	1	-0.0536	0.3804	-0.1208	0.2702	-0.2732	0.3132	2	-0.0158	0.5068	-0.1145	0.3234	-0.2694	0.3272
		300	1	-0.0331	0.3107	-0.0873	0.1894	-0.2275	0.2520	2	0.0007	0.4163	-0.0743	0.2312	-0.2237	0.2601
		500	1	-0.0155	0.2444	-0.0630	0.1522	-0.2031	0.2192	2	0.0021	0.3210	-0.0560	0.1873	-0.1993	0.2236
		1000	1	-0.0065	0.1995	-0.0384	0.1137	-0.1689	0.1797	2	0.0098	0.2523	-0.0323	0.1350	-0.1662	0.1829
	1.1	150	1	-0.0425	0.3908	-0.1300	0.2692	-0.2736	0.3121	2	-0.0358	0.4928	-0.1256	0.3275	-0.2696	0.3288
		300	1	-0.0147	0.3079	-0.0877	0.1918	-0.2283	0.2514	2	-0.0043	0.4237	-0.0819	0.2355	-0.2294	0.2672
		500	1	-0.0119	0.2459	-0.0612	0.1487	-0.2039	0.2210	2	0.0015	0.3303	-0.0547	0.1810	-0.2033	0.2283
		1000	1	-0.0053	0.2028	-0.0376	0.1132	-0.1677	0.1786	2	-0.0024	0.2637	-0.0329	0.1369	-0.1669	0.1826
	1.3	150	1	-0.0397	0.3908	-0.1190	0.2623	-0.2679	0.3066	2	-0.0622	0.5196	-0.1242	0.3281	-0.2747	0.3295
		300	1	-0.0123	0.3119	-0.0834	0.1901	-0.2260	0.2497	2	-0.0187	0.4131	-0.0762	0.2254	-0.2278	0.2641
		500	1	-0.0062	0.2472	-0.0574	0.1513	-0.2009	0.2176	2	0.0019	0.3272	-0.0590	0.1880	-0.2059	0.2317
		1000	1	-0.0036	0.2018	-0.0344	0.1099	-0.1676	0.1788	2	-0.0021	0.2503	-0.0386	0.1387	-0.1682	0.1846
	1.5	150	2	-0.0540	0.5106	-0.1281	0.3242	-0.2828	0.3381	3	0.0027	0.6139	-0.0922	0.3736	-0.2515	0.3370
		300	2	-0.0214	0.4234	-0.0866	0.2338	-0.2300	0.2660	3	0.0354	0.4840	-0.0607	0.2582	-0.2119	0.2636
		500	2	-0.0111	0.3275	-0.0586	0.1870	-0.2045	0.2288	3	0.0372	0.4115	-0.0439	0.2077	-0.1951	0.2291
		1000	2	-0.0066	0.2538	-0.0424	0.1391	-0.1684	0.1847	3	0.0318	0.3244	-0.0266	0.1566	-0.1633	0.1856
	1.7	150	2	-0.0553	0.4963	-0.1398	0.3300	-0.2803	0.3383	3	-0.0031	0.6377	-0.1158	0.3792	-0.2594	0.3456
		300	2	-0.0205	0.4144	-0.0922	0.2335	-0.2309	0.2658	3	0.0104	0.4856	-0.0668	0.2617	-0.2169	0.2660
		500	2	-0.0142	0.3345	-0.0605	0.1850	-0.2048	0.2296	3	0.0323	0.4080	-0.0481	0.2108	-0.2006	0.2337
		1000	2	-0.0064	0.2555	-0.0425	0.1414	-0.1715	0.1873	3	0.0267	0.3151	-0.0276	0.1567	-0.1631	0.1863
	1.9	150	2	-0.0501	0.4944	-0.1393	0.3355	-0.2831	0.3397	3	-0.0352	0.6361	-0.1168	0.3789	-0.2671	0.3475
		300	2	-0.0308	0.4310	-0.0912	0.2363	-0.2298	0.2661	3	-0.0003	0.4874	-0.0806	0.2661	-0.2256	0.2745
		500	2	-0.0149	0.3264	-0.0632	0.1866	-0.2054	0.2303	3	-0.0032	0.4081	-0.0503	0.2067	-0.2017	0.2357
		1000	2	-0.0108	0.2600	-0.0383	0.1366	-0.1674	0.1838	3	0.0049	0.3133	-0.0328	0.1586	-0.1649	0.1872

[a] The data generation is $(1-L)^d X_t = (1-.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 10: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$ and $p = 3$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	1	-0.0631	0.3633	-0.1425	0.2498	-0.2822	0.3085	2	0.0423	0.6023	-0.0940	0.2716	-0.2591	0.3037
		300	1	-0.0222	0.2770	-0.0953	0.1692	-0.2330	0.2482	2	0.0590	0.3884	-0.0599	0.1944	-0.2191	0.2442
		500	1	-0.0186	0.2303	-0.0653	0.1315	-0.2084	0.2186	2	0.0597	0.3121	-0.0313	0.1501	-0.1876	0.2067
		1000	1	-0.0082	0.1774	-0.0399	0.0949	-0.1711	0.1780	2	0.0372	0.2276	-0.0155	0.1127	-0.1606	0.1716
	.7	150	1	-0.0680	0.3532	-0.1497	0.2504	-0.2853	0.3109	2	0.0132	0.5854	-0.1120	0.2725	-0.2759	0.3164
		300	1	-0.0295	0.2713	-0.0963	0.1677	-0.2345	0.2502	2	0.0275	0.3861	-0.0692	0.1954	-0.2279	0.2518
		500	1	-0.0171	0.2245	-0.0688	0.1330	-0.2094	0.2198	2	0.0282	0.3001	-0.0421	0.1527	-0.1918	0.2095
		1000	1	-0.0112	0.1767	-0.0417	0.0948	-0.1696	0.1762	2	0.0295	0.2186	-0.0221	0.1145	-0.1649	0.1762
	.9	150	1	-0.0688	0.3532	-0.1426	0.2433	-0.2796	0.3051	2	-0.0054	0.5824	-0.1203	0.2777	-0.2829	0.3229
		300	1	-0.0305	0.2658	-0.0963	0.1679	-0.2318	0.2475	2	0.0019	0.3822	-0.0751	0.1933	-0.2304	0.2542
		500	1	-0.0161	0.2212	-0.0677	0.1316	-0.2082	0.2185	2	0.0170	0.3032	-0.0528	0.1553	-0.1967	0.2135
		1000	1	-0.0080	0.1723	-0.0398	0.0938	-0.1706	0.1773	2	0.0151	0.2184	-0.0250	0.1142	-0.1671	0.1778
	1.1	150	1	-0.0625	0.3495	-0.1345	0.2382	-0.2802	0.3057	2	-0.0272	0.5816	-0.1412	0.2886	-0.2882	0.3266
		300	1	-0.0197	0.2650	-0.0896	0.1630	-0.2299	0.2452	2	-0.0053	0.3765	-0.0831	0.1979	-0.2344	0.2578
		500	1	-0.0056	0.2197	-0.0613	0.1278	-0.2055	0.2162	2	0.0030	0.2954	-0.0573	0.1561	-0.2029	0.2193
		1000	1	-0.0003	0.1750	-0.0380	0.0909	-0.1700	0.1768	2	-0.0035	0.2149	-0.0356	0.1155	-0.1704	0.1807
	1.3	150	1	-0.0192	0.3432	-0.1166	0.2299	-0.2710	0.2979	2	-0.0308	0.5784	-0.1461	0.2887	-0.2987	0.3356
		300	1	0.0005	0.2701	-0.0786	0.1606	-0.2234	0.2393	2	-0.0260	0.3751	-0.0930	0.2016	-0.2415	0.2628
		500	1	0.0121	0.2211	-0.0530	0.1242	-0.2019	0.2126	2	-0.0129	0.2938	-0.0667	0.1594	-0.2066	0.2228
		1000	1	0.0160	0.1726	-0.0315	0.0905	-0.1672	0.1742	2	-0.0063	0.2144	-0.0386	0.1149	-0.1725	0.1822
	1.5	150	2	-0.0629	0.5596	-0.1559	0.2952	-0.3025	0.3388	3	0.0207	0.6073	-0.0929	0.3273	-0.2688	0.3267
		300	2	-0.0287	0.3722	-0.0985	0.1985	-0.2424	0.2634	3	0.0826	0.6085	-0.0432	0.2363	-0.2136	0.2516
		500	2	-0.0100	0.2845	-0.0645	0.1536	-0.2065	0.2227	3	0.0523	0.3918	-0.0350	0.1769	-0.1952	0.2175
		1000	2	-0.0147	0.2114	-0.0379	0.1136	-0.1731	0.1835	3	0.0425	0.2655	-0.0131	0.1341	-0.1595	0.1749
	1.7	150	2	-0.0688	0.5600	-0.1574	0.2919	-0.3039	0.3380	3	-0.0077	0.6090	-0.1098	0.3368	-0.2779	0.3342
		300	2	-0.0309	0.3733	-0.1001	0.2004	-0.2433	0.2653	3	0.0588	0.6032	-0.0557	0.2376	-0.2212	0.2570
		500	2	-0.0197	0.2876	-0.0701	0.1584	-0.2077	0.2238	3	0.0385	0.3906	-0.0407	0.1807	-0.2036	0.2265
		1000	2	-0.0129	0.2138	-0.0421	0.1155	-0.1742	0.1842	3	0.0288	0.2617	-0.0233	0.1319	-0.1645	0.1791
	1.9	150	2	-0.0719	0.5697	-0.1520	0.2840	-0.2998	0.3346	3	-0.0313	0.6004	-0.1255	0.3392	-0.2886	0.3418
		300	2	-0.0372	0.3691	-0.0993	0.1994	-0.2409	0.2625	3	0.0064	0.5973	-0.0694	0.2382	-0.2275	0.2610
		500	2	-0.0146	0.2860	-0.0682	0.1536	-0.2062	0.2223	3	0.0140	0.3991	-0.0557	0.1796	-0.2066	0.2282
		1000	2	-0.0070	0.2108	-0.0408	0.1154	-0.1711	0.1810	3	0.0176	0.2579	-0.0274	0.1298	-0.1658	0.1802

[a] The data generation is $(1-L)^d X_t = (1-.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;

[b] The p is the pooling order;

[c] The τ is the tapering order;

[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;

[e] The m is the number of the frequency ordinates involved in the estimation.

Table 11: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .4$ and $p = 1$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	1	0.0111	0.3742	0.0312	0.2369	0.1584	0.2174	2	0.0880	0.5023	0.0767	0.3068	0.1808	0.2587
		300	1	-0.0040	0.3119	0.0208	0.1699	0.1103	0.1529	2	0.0636	0.4365	0.0537	0.2219	0.1261	0.1821
		500	1	0.0027	0.2487	0.0137	0.1370	0.0861	0.1206	2	0.0393	0.3274	0.0358	0.1743	0.1004	0.1449
		1000	1	0.0065	0.2022	0.0079	0.1083	0.0619	0.0871	2	0.0359	0.2522	0.0242	0.1368	0.0682	0.1014
	.7	150	1	0.0038	0.3754	0.0341	0.2384	0.1554	0.2157	2	0.0545	0.4978	0.0642	0.3121	0.1765	0.2563
		300	1	0.0038	0.3133	0.0218	0.1733	0.1079	0.1528	2	0.0442	0.4092	0.0335	0.2220	0.1175	0.1784
		500	1	0.0053	0.2489	0.0139	0.1392	0.0887	0.1223	2	0.0325	0.3275	0.0313	0.1784	0.0993	0.1427
		1000	1	-0.0034	0.2010	0.0083	0.1094	0.0628	0.0876	2	0.0285	0.2544	0.0178	0.1348	0.0672	0.1016
	.9	150	1	0.0029	0.3822	0.0377	0.2348	0.1527	0.2144	2	0.0506	0.5101	0.0494	0.3045	0.1704	0.2541
		300	1	0.0002	0.3126	0.0225	0.1714	0.1122	0.1539	2	0.0254	0.4168	0.0359	0.2189	0.1150	0.1760
		500	1	0.0050	0.2454	0.0123	0.1374	0.0877	0.1209	2	0.0298	0.3206	0.0218	0.1784	0.0941	0.1393
		1000	1	0.0005	0.1978	0.0098	0.1075	0.0648	0.0895	2	0.0178	0.2552	0.0147	0.1343	0.0670	0.1000
	1.1	150	1	0.0158	0.3848	0.0362	0.2351	0.1525	0.2149	2	0.0335	0.4961	0.0506	0.3007	0.1643	0.2487
		300	1	0.0039	0.3061	0.0202	0.1651	0.1114	0.1532	2	0.0269	0.4200	0.0291	0.2169	0.1128	0.1727
		500	1	0.0044	0.2467	0.0175	0.1418	0.0867	0.1206	2	0.0206	0.3192	0.0252	0.1745	0.0941	0.1383
		1000	1	0.0053	0.2003	0.0064	0.1080	0.0630	0.0884	2	0.0100	0.2546	0.0100	0.1350	0.0622	0.0994
	1.3	150	1	0.0178	0.3782	0.0482	0.2391	0.1606	0.2201	2	0.0277	0.4994	0.0371	0.3055	0.1624	0.2495
		300	1	0.0152	0.3044	0.0245	0.1750	0.1098	0.1544	2	0.0153	0.4163	0.0275	0.2174	0.1129	0.1737
		500	1	0.0111	0.2484	0.0183	0.1408	0.0898	0.1221	2	0.0095	0.3317	0.0138	0.1766	0.0891	0.1374
		1000	1	0.0084	0.2034	0.0101	0.1066	0.0640	0.0887	2	0.0073	0.2556	0.0133	0.1332	0.0640	0.0980
	1.5	150	2	0.0090	0.5011	0.0356	0.3040	0.1618	0.2464	3	0.0862	0.6375	0.0775	0.3635	0.1792	0.2889
		300	2	-0.0003	0.4162	0.0262	0.2195	0.1079	0.1714	3	0.0619	0.4856	0.0518	0.2604	0.1207	0.1957
		500	2	0.0014	0.3253	0.0148	0.1780	0.0878	0.1341	3	0.0411	0.4034	0.0394	0.2042	0.1028	0.1567
		1000	2	-0.0031	0.2553	0.0101	0.1315	0.0638	0.1008	3	0.0423	0.3194	0.0255	0.1540	0.0704	0.1127
	1.7	150	2	0.0024	0.5041	0.0283	0.3078	0.1598	0.2448	3	0.0499	0.6280	0.0642	0.3643	0.1643	0.2788
		300	2	0.0042	0.4238	0.0180	0.2161	0.1073	0.1690	3	0.0564	0.4911	0.0472	0.2539	0.1200	0.1955
		500	2	0.0010	0.3200	0.0116	0.1769	0.0882	0.1357	3	0.0340	0.4086	0.0333	0.2028	0.0985	0.1547
		1000	2	-0.0006	0.2555	0.0055	0.1302	0.0641	0.0997	3	0.0349	0.3242	0.0186	0.1584	0.0672	0.1113
	1.9	150	2	0.0028	0.4990	0.0348	0.3041	0.1576	0.2441	3	0.0528	0.6286	0.0583	0.3639	0.1652	0.2774
		300	2	0.0077	0.4180	0.0167	0.2168	0.1110	0.1715	3	0.0373	0.4835	0.0358	0.2552	0.1133	0.1935
		500	2	0.0039	0.3206	0.0082	0.1759	0.0868	0.1352	3	0.0262	0.4066	0.0231	0.2036	0.0927	0.1518
		1000	2	0.0001	0.2530	0.0082	0.1346	0.0614	0.0983	3	0.0175	0.3212	0.0148	0.1554	0.0671	0.1115

[a] The data is generated by $(1-L)^d X_t = (1+.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 12: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .4$ and $p = 3$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	1	-0.0058	0.3525	0.0320	0.2005	0.1500	0.1957	2	0.1183	0.6126	0.1019	0.2751	0.2141	0.2659
		300	1	0.0033	0.2745	0.0189	0.1424	0.1055	0.1366	2	0.0848	0.3973	0.0692	0.1943	0.1419	0.1801
		500	1	-0.0047	0.2311	0.0112	0.1139	0.0876	0.1103	2	0.0714	0.3110	0.0471	0.1528	0.1022	0.1336
		1000	1	0.0009	0.1787	0.0055	0.0859	0.0622	0.0791	2	0.0486	0.2262	0.0322	0.1192	0.0745	0.0966
	.7	150	1	0.0002	0.3422	0.0343	0.1974	0.1495	0.1946	2	0.0727	0.5924	0.0880	0.2750	0.1967	0.2509
		300	1	-0.0098	0.2677	0.0199	0.1400	0.1046	0.1362	2	0.0493	0.3887	0.0570	0.1910	0.1341	0.1723
		500	1	-0.0022	0.2268	0.0116	0.1131	0.0882	0.1109	2	0.0498	0.3110	0.0389	0.1509	0.0962	0.1275
		1000	1	-0.0098	0.1791	0.0067	0.0856	0.0612	0.0782	2	0.0303	0.2257	0.0226	0.1121	0.0713	0.0942
	.9	150	1	0.0005	0.3432	0.0314	0.2002	0.1530	0.1956	2	0.0604	0.5725	0.0713	0.2657	0.1908	0.2479
		300	1	-0.0031	0.2687	0.0211	0.1399	0.1052	0.1358	2	0.0475	0.3769	0.0410	0.1857	0.1258	0.1660
		500	1	-0.0027	0.2288	0.0130	0.1117	0.0866	0.1097	2	0.0383	0.2993	0.0320	0.1457	0.0935	0.1260
		1000	1	-0.0076	0.1726	0.0066	0.0861	0.0626	0.0790	2	0.0263	0.2140	0.0176	0.1120	0.0682	0.0907
	1.1	150	1	0.0263	0.3423	0.0437	0.2055	0.1565	0.1986	2	0.0266	0.5685	0.0504	0.2552	0.1814	0.2378
		300	1	0.0087	0.2626	0.0240	0.1402	0.1082	0.1378	2	0.0153	0.3792	0.0349	0.1794	0.1191	0.1602
		500	1	0.0042	0.2200	0.0166	0.1118	0.0907	0.1114	2	0.0228	0.2967	0.0183	0.1462	0.0871	0.1201
		1000	1	0.0057	0.1711	0.0104	0.0850	0.0637	0.0799	2	0.0111	0.2156	0.0137	0.1105	0.0656	0.0889
	1.3	150	1	0.0514	0.3432	0.0548	0.2053	0.1655	0.2055	2	0.0166	0.5652	0.0402	0.2542	0.1763	0.2337
		300	1	0.0286	0.2670	0.0359	0.1412	0.1157	0.1438	2	0.0008	0.3756	0.0218	0.1815	0.1181	0.1586
		500	1	0.0200	0.2226	0.0253	0.1127	0.0946	0.1152	2	0.0056	0.2938	0.0150	0.1433	0.0883	0.1219
		1000	1	0.0201	0.1779	0.0151	0.0860	0.0659	0.0817	2	0.0030	0.2131	0.0097	0.1095	0.0649	0.0884
	1.5	150	2	0.0105	0.5628	0.0387	0.2475	0.1698	0.2285	3	0.0897	0.6100	0.0905	0.3332	0.2069	0.2775
		300	2	0.0120	0.3692	0.0252	0.1762	0.1155	0.1554	3	0.0811	0.6079	0.0639	0.2409	0.1304	0.1861
		500	2	-0.0030	0.2953	0.0131	0.1409	0.0843	0.1191	3	0.0580	0.4003	0.0440	0.1782	0.1077	0.1446
		1000	2	0.0012	0.2156	0.0054	0.1102	0.0659	0.0883	3	0.0556	0.2705	0.0313	0.1374	0.0736	0.1037
	1.7	150	2	0.0155	0.5603	0.0359	0.2480	0.1684	0.2264	3	0.0705	0.6148	0.0880	0.3259	0.2021	0.2735
		300	2	-0.0026	0.3670	0.0171	0.1791	0.1130	0.1537	3	0.0490	0.6071	0.0476	0.2372	0.1204	0.1753
		500	2	-0.0058	0.2891	0.0124	0.1443	0.0837	0.1175	3	0.0490	0.3998	0.0356	0.1751	0.1027	0.1413
		1000	2	0.0007	0.2074	0.0069	0.1079	0.0634	0.0867	3	0.0361	0.2641	0.0240	0.1333	0.0700	0.1002
	1.9	150	2	0.0060	0.5558	0.0396	0.2490	0.1755	0.2306	3	0.0558	0.5911	0.0610	0.3226	0.1882	0.2605
		300	2	0.0051	0.3664	0.0190	0.1733	0.1158	0.1550	3	0.0345	0.5937	0.0432	0.2325	0.1161	0.1751
		500	2	-0.0031	0.2870	0.0129	0.1397	0.0858	0.1191	3	0.0288	0.3902	0.0271	0.1728	0.0975	0.1375
		1000	2	0.0000	0.2080	0.0033	0.1066	0.0649	0.0880	3	0.0226	0.2547	0.0181	0.1323	0.0678	0.0981

[a] The data is generated by $(1-L)^d X_t = (1+.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[d] The m is the number of the frequency ordinates involved in the estimation;
[e] The τ is the tapering order.

Table 13: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$ and $p = 1$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	1	0.0084	0.3849	0.0368	0.2393	0.2177	0.2649	2	0.0645	0.5069	0.0878	0.3147	0.2500	0.3101
		300	1	0.0034	0.3099	0.0264	0.1748	0.1438	0.1786	2	0.0604	0.4237	0.0538	0.2224	0.1587	0.2067
		500	1	0.0005	0.2461	0.0186	0.1410	0.1129	0.1412	2	0.0466	0.3302	0.0424	0.1814	0.1240	0.1620
		1000	1	0.0010	0.2018	0.0097	0.1070	0.0790	0.1012	2	0.0347	0.2530	0.0229	0.1337	0.0865	0.1160
	.7	150	1	0.0179	0.3945	0.0397	0.2422	0.2154	0.2611	2	0.0674	0.5011	0.0725	0.3064	0.2416	0.3025
		300	1	0.0028	0.3096	0.0257	0.1725	0.1420	0.1767	2	0.0457	0.4274	0.0511	0.2201	0.1534	0.2042
		500	1	0.0042	0.2495	0.0130	0.1421	0.1098	0.1387	2	0.0333	0.3247	0.0340	0.1791	0.1228	0.1610
		1000	1	-0.0039	0.2027	0.0112	0.1084	0.0791	0.0995	2	0.0244	0.2617	0.0236	0.1361	0.0850	0.1138
	.9	150	1	0.0029	0.3928	0.0372	0.2413	0.2130	0.2618	2	0.0512	0.5019	0.0619	0.3103	0.2330	0.3001
		300	1	0.0012	0.3137	0.0256	0.1706	0.1433	0.1796	2	0.0290	0.4165	0.0343	0.2169	0.1502	0.2010
		500	1	0.0042	0.2471	0.0182	0.1392	0.1125	0.1397	2	0.0215	0.3236	0.0233	0.1808	0.1184	0.1555
		1000	1	0.0028	0.2013	0.0102	0.1085	0.0807	0.1003	2	0.0216	0.2534	0.0170	0.1331	0.0827	0.1126
	1.1	150	1	0.0172	0.3772	0.0463	0.2443	0.2164	0.2639	2	0.0303	0.5030	0.0524	0.3060	0.2286	0.2957
		300	1	0.0086	0.3114	0.0300	0.1756	0.1440	0.1800	2	0.0222	0.4038	0.0362	0.2173	0.1474	0.1984
		500	1	0.0103	0.2429	0.0200	0.1398	0.1147	0.1418	2	0.0219	0.3236	0.0264	0.1760	0.1167	0.1553
		1000	1	0.0084	0.1990	0.0122	0.1085	0.0799	0.1004	2	0.0086	0.2550	0.0120	0.1330	0.0818	0.1115
	1.3	150	1	0.0318	0.3831	0.0518	0.2431	0.2201	0.2660	2	0.0166	0.5013	0.0487	0.3064	0.2265	0.2964
		300	1	0.0291	0.3098	0.0350	0.1737	0.1484	0.1825	2	0.0123	0.4164	0.0305	0.2176	0.1441	0.1953
		500	1	0.0158	0.2487	0.0182	0.1382	0.1157	0.1421	2	0.0054	0.3302	0.0189	0.1788	0.1131	0.1545
		1000	1	0.0056	0.2002	0.0121	0.1087	0.0808	0.1013	2	0.0065	0.2560	0.0094	0.1331	0.0781	0.1088
	1.5	150	2	0.0007	0.5042	0.0460	0.3042	0.2244	0.2924	3	0.0815	0.6363	0.0860	0.3660	0.2348	0.3242
		300	2	0.0079	0.4187	0.0263	0.2179	0.1425	0.1946	3	0.0583	0.4901	0.0547	0.2544	0.1596	0.2236
		500	2	0.0070	0.3223	0.0159	0.1805	0.1169	0.1551	3	0.0493	0.4051	0.0461	0.2066	0.1288	0.1761
		1000	2	0.0024	0.2524	0.0118	0.1328	0.0799	0.1105	3	0.0396	0.3219	0.0257	0.1602	0.0845	0.1224
	1.7	150	2	0.0025	0.4953	0.0451	0.3035	0.2207	0.2876	3	0.0626	0.6288	0.0711	0.3685	0.2222	0.3153
		300	2	0.0067	0.4274	0.0266	0.2171	0.1399	0.1951	3	0.0525	0.4851	0.0459	0.2576	0.1512	0.2186
		500	2	-0.0033	0.3240	0.0125	0.1768	0.1156	0.1548	3	0.0299	0.4153	0.0345	0.2060	0.1229	0.1702
		1000	2	0.0026	0.2518	0.0070	0.1349	0.0788	0.1082	3	0.0339	0.3110	0.0243	0.1569	0.0872	0.1241
	1.9	150	2	0.0032	0.5029	0.0450	0.3002	0.2231	0.2909	3	0.0455	0.6352	0.0654	0.3657	0.2138	0.3106
		300	2	0.0036	0.4237	0.0242	0.2199	0.1405	0.1941	3	0.0368	0.4796	0.0366	0.2517	0.1513	0.2184
		500	2	0.0057	0.3309	0.0155	0.1800	0.1111	0.1519	3	0.0258	0.4040	0.0305	0.2054	0.1228	0.1714
		1000	2	-0.0065	0.2522	0.0109	0.1313	0.0788	0.1089	3	0.0175	0.3124	0.0153	0.1565	0.0839	0.1220

[a] The data is generated by $(1-L)^d X_t = (1+.8L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 14: Monte Carlo Results of the GPHF Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$ and $p = 3$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	1	0.0168	0.3514	0.0443	0.2071	0.2061	0.2428	2	0.0933	0.6005	0.1080	0.2803	0.2790	0.3198
		300	1	-0.0035	0.2729	0.0285	0.1431	0.1390	0.1648	2	0.0901	0.4004	0.0721	0.1979	0.1767	0.2082
		500	1	0.0010	0.2281	0.0171	0.1126	0.1146	0.1324	2	0.0773	0.3122	0.0513	0.1557	0.1279	0.1539
		1000	1	0.0016	0.1821	0.0113	0.0878	0.0788	0.0924	2	0.0528	0.2249	0.0306	0.1160	0.0920	0.1107
	.7	150	1	0.0079	0.3472	0.0432	0.2008	0.2075	0.2422	2	0.0889	0.5887	0.0918	0.2747	0.2674	0.3100
		300	1	-0.0052	0.2679	0.0202	0.1395	0.1384	0.1629	2	0.0707	0.3950	0.0569	0.1922	0.1692	0.2013
		500	1	-0.0032	0.2220	0.0156	0.1128	0.1137	0.1327	2	0.0498	0.3060	0.0434	0.1497	0.1222	0.1491
		1000	1	-0.0043	0.1781	0.0103	0.0846	0.0791	0.0927	2	0.0322	0.2233	0.0264	0.1142	0.0897	0.1080
	.9	150	1	0.0101	0.3417	0.0474	0.2045	0.2076	0.2416	2	0.0558	0.5812	0.0706	0.2594	0.2582	0.3009
		300	1	-0.0051	0.2679	0.0231	0.1391	0.1408	0.1643	2	0.0305	0.3796	0.0431	0.1865	0.1655	0.1970
		500	1	0.0017	0.2206	0.0190	0.1109	0.1143	0.1319	2	0.0242	0.2953	0.0309	0.1476	0.1189	0.1463
		1000	1	0.0035	0.1741	0.0098	0.0842	0.0792	0.0931	2	0.0245	0.2198	0.0204	0.1113	0.0848	0.1048
	1.1	150	1	0.0196	0.3426	0.0498	0.2061	0.2122	0.2457	2	0.0451	0.5740	0.0690	0.2587	0.2517	0.2936
		300	1	0.0116	0.2651	0.0306	0.1423	0.1439	0.1670	2	0.0334	0.3779	0.0380	0.1827	0.1602	0.1915
		500	1	0.0107	0.2183	0.0210	0.1145	0.1151	0.1326	2	0.0120	0.2970	0.0262	0.1467	0.1136	0.1407
		1000	1	0.0061	0.1733	0.0135	0.0849	0.0816	0.0944	2	0.0090	0.2152	0.0151	0.1082	0.0827	0.1023
	1.3	150	1	0.0547	0.3441	0.0676	0.2101	0.2198	0.2514	2	0.0100	0.5677	0.0504	0.2523	0.2447	0.2895
		300	1	0.0360	0.2670	0.0447	0.1444	0.1490	0.1722	2	0.0091	0.3801	0.0298	0.1810	0.1557	0.1878
		500	1	0.0260	0.2235	0.0286	0.1162	0.1218	0.1382	2	0.0041	0.2899	0.0238	0.1434	0.1115	0.1394
		1000	1	0.0188	0.1755	0.0176	0.0870	0.0832	0.0959	2	0.0013	0.2162	0.0125	0.1083	0.0806	0.1004
	1.5	150	2	0.0086	0.5669	0.0485	0.2531	0.2400	0.2838	3	0.0976	0.6105	0.1088	0.3261	0.2728	0.3312
		300	2	0.0065	0.3687	0.0257	0.1776	0.1567	0.1878	3	0.0929	0.6114	0.0692	0.2393	0.1641	0.2087
		500	2	0.0080	0.2896	0.0164	0.1428	0.1091	0.1369	3	0.0789	0.4081	0.0510	0.1828	0.1341	0.1663
		1000	2	0.0021	0.2115	0.0105	0.1085	0.0814	0.1014	3	0.0530	0.2676	0.0320	0.1359	0.0915	0.1158
	1.7	150	2	0.0031	0.5630	0.0442	0.2456	0.2385	0.2811	3	0.0809	0.5993	0.0877	0.3265	0.2661	0.3252
		300	2	-0.0046	0.3724	0.0240	0.1770	0.1522	0.1852	3	0.0647	0.5923	0.0543	0.2364	0.1555	0.2020
		500	2	-0.0053	0.2891	0.0155	0.1450	0.1084	0.1354	3	0.0471	0.3943	0.0343	0.1772	0.1289	0.1619
		1000	2	-0.0045	0.2099	0.0074	0.1086	0.0822	0.1011	3	0.0282	0.2617	0.0249	0.1327	0.0870	0.1128
	1.9	150	2	0.0118	0.5558	0.0546	0.2502	0.2391	0.2825	3	0.0430	0.6007	0.0621	0.3253	0.2537	0.3116
		300	2	-0.0026	0.3672	0.0280	0.1757	0.1527	0.1860	3	0.0318	0.5943	0.0405	0.2354	0.1474	0.1978
		500	2	0.0019	0.2818	0.0150	0.1414	0.1093	0.1372	3	0.0357	0.3929	0.0339	0.1734	0.1264	0.1597
		1000	2	0.0004	0.2090	0.0074	0.1078	0.0810	0.1004	3	0.0236	0.2620	0.0198	0.1301	0.0840	0.1097

[a] The data is generated by $(1-L)^d X_t = (1+.8L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The p is the pooling order;
[c] The τ is the tapering order;
[d] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[e] The m is the number of the frequency ordinates involved in the estimation.

Table 15: Monte Carlo Results of the Tapered Local Whittle Estimation for ARFIMA$(0, d, 0)$ Model

			$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
			$m=\lfloor n^{.5} \rfloor$		$m=\lfloor n^{.65} \rfloor$		$m=\lfloor n^{.8} \rfloor$			$m=\lfloor n^{.5} \rfloor$		$m=\lfloor n^{.65} \rfloor$		$m=\lfloor n^{.8} \rfloor$	
d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	1	-0.0271	0.2918	-0.0180	0.1715	-0.0110	0.1088	2	0.0173	0.3553	0.0135	0.2025	0.0038	0.1278
	300	1	-0.0207	0.2203	-0.0122	0.1237	-0.0061	0.0750	2	0.0114	0.2698	0.0089	0.1466	0.0035	0.0887
	500	1	-0.0161	0.1849	-0.0088	0.0985	-0.0023	0.0586	2	0.0081	0.2253	0.0047	0.1177	0.0024	0.0686
	1000	1	-0.0170	0.1447	-0.0079	0.0768	-0.0017	0.0421	2	0.0052	0.1744	0.0044	0.0886	0.0024	0.0491
.7	150	1	-0.0318	0.2876	-0.0146	0.1688	-0.0100	0.1076	2	0.0053	0.3557	0.0062	0.2073	-0.0041	0.1276
	300	1	-0.0211	0.2211	-0.0096	0.1219	-0.0070	0.0762	2	0.0009	0.2714	0.0037	0.1472	-0.0012	0.0886
	500	1	-0.0178	0.1856	-0.0079	0.1016	-0.0032	0.0593	2	0.0019	0.2210	0.0031	0.1184	-0.0008	0.0692
	1000	1	-0.0142	0.1469	-0.0056	0.0760	-0.0034	0.0428	2	0.0043	0.1729	-0.0020	0.0864	0.0006	0.0498
.9	150	1	-0.0335	0.2824	-0.0145	0.1718	-0.0102	0.1073	2	-0.0074	0.3616	-0.0036	0.2052	-0.0073	0.1272
	300	1	-0.0215	0.2180	-0.0095	0.1214	-0.0076	0.0762	2	-0.0076	0.2689	-0.0023	0.1477	-0.0033	0.0886
	500	1	-0.0193	0.1833	-0.0094	0.0983	-0.0050	0.0573	2	-0.0080	0.2223	0.0003	0.1146	-0.0024	0.0674
	1000	1	-0.0108	0.1429	-0.0057	0.0753	-0.0018	0.0427	2	-0.0043	0.1763	-0.0029	0.0869	-0.0010	0.0495
1.1	150	1	-0.0224	0.2932	-0.0148	0.1694	-0.0089	0.1073	2	-0.0189	0.3657	-0.0137	0.2081	-0.0079	0.1274
	300	1	-0.0148	0.2212	-0.0127	0.1219	-0.0057	0.0766	2	-0.0173	0.2716	-0.0067	0.1441	-0.0053	0.0878
	500	1	-0.0138	0.1848	-0.0068	0.0987	-0.0060	0.0583	2	-0.0143	0.2208	-0.0081	0.1190	-0.0028	0.0703
	1000	1	-0.0129	0.1456	-0.0057	0.0749	-0.0037	0.0423	2	-0.0078	0.1772	-0.0032	0.0869	-0.0031	0.0494
1.3	150	1	-0.0135	0.2852	-0.0080	0.1691	-0.0078	0.1067	2	-0.0300	0.3634	-0.0196	0.2096	-0.0127	0.1283
	300	1	-0.0106	0.2191	-0.0082	0.1237	-0.0033	0.0750	2	-0.0229	0.2707	-0.0128	0.1497	-0.0083	0.0892
	500	1	-0.0114	0.1816	-0.0053	0.0997	-0.0022	0.0573	2	-0.0144	0.2214	-0.0095	0.1194	-0.0047	0.0686
	1000	1	-0.0065	0.1435	-0.0050	0.0742	-0.0014	0.0425	2	-0.0115	0.1756	-0.0043	0.0878	-0.0023	0.0495
1.5	150	2	-0.0338	0.3710	-0.0210	0.2084	-0.0127	0.1268	3	0.0210	0.4259	-0.0002	0.2429	-0.0011	0.1461
	300	2	-0.0231	0.2714	-0.0096	0.1464	-0.0065	0.0883	3	0.0183	0.3142	0.0040	0.1699	0.0010	0.1009
	500	2	-0.0176	0.2256	-0.0089	0.1190	-0.0053	0.0681	3	0.0120	0.2541	0.0010	0.1327	0.0013	0.0758
	1000	2	-0.0159	0.1724	-0.0054	0.0878	-0.0036	0.0491	3	0.0022	0.2000	0.0022	0.0994	0.0013	0.0545
1.7	150	2	-0.0280	0.3630	-0.0254	0.2116	-0.0124	0.1292	3	0.0018	0.4249	0.0051	0.2383	0.0004	0.1463
	300	2	-0.0297	0.2759	-0.0135	0.1476	-0.0084	0.0890	3	0.0023	0.3242	-0.0012	0.1677	-0.0006	0.0998
	500	2	-0.0270	0.2254	-0.0103	0.1183	-0.0057	0.0682	3	-0.0006	0.2628	-0.0027	0.1349	-0.0014	0.0761
	1000	2	-0.0178	0.1740	-0.0085	0.0878	-0.0029	0.0495	3	0.0028	0.2023	-0.0007	0.0988	0.0002	0.0552
1.9	150	2	-0.0265	0.3616	-0.0166	0.2034	-0.0118	0.1267	3	-0.0081	0.4251	-0.0095	0.2434	-0.0060	0.1465
	300	2	-0.0279	0.2666	-0.0142	0.1466	-0.0092	0.0888	3	-0.0104	0.3172	-0.0014	0.1696	-0.0053	0.1014
	500	2	-0.0233	0.2274	-0.0141	0.1171	-0.0034	0.0671	3	-0.0089	0.2545	-0.0033	0.1334	-0.0051	0.0789
	1000	2	-0.0124	0.1751	-0.0067	0.0887	-0.0037	0.0494	3	-0.0074	0.2023	-0.0036	0.1011	-0.0013	0.0537

[a] The data generation is $(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[c] The m is the number of the frequency ordinates involved in the estimation;
[d] The τ is the tapering order.

Table 16: Monte Carlo Results of the Tapered Local Whittle Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	1	0.0292	0.2975	0.1350	0.2194	0.3050	0.3244	2	0.0939	0.3705	0.1795	0.2766	0.3354	0.3622
		300	1	0.0037	0.2159	0.0886	0.1531	0.2518	0.2642	2	0.0448	0.2699	0.1102	0.1857	0.2683	0.2837
		500	1	0.0050	0.1822	0.0624	0.1186	0.2221	0.2306	2	0.0283	0.2195	0.0746	0.1412	0.2322	0.2429
		1000	1	-0.0083	0.1454	0.0379	0.0833	0.1819	0.1874	2	0.0179	0.1761	0.0452	0.0997	0.1897	0.1962
	.7	150	1	0.0205	0.2879	0.1355	0.2203	0.3011	0.3218	2	0.0752	0.3719	0.1694	0.2685	0.3288	0.3541
		300	1	-0.0005	0.2168	0.0868	0.1516	0.2513	0.2641	2	0.0344	0.2739	0.1079	0.1830	0.2710	0.2866
		500	1	-0.0027	0.1873	0.0587	0.1162	0.2216	0.2299	2	0.0213	0.2241	0.0732	0.1383	0.2315	0.2425
		1000	1	-0.0083	0.1437	0.0359	0.0836	0.1825	0.1877	2	0.0118	0.1716	0.0460	0.0993	0.1898	0.1964
	.9	150	1	0.0279	0.2954	0.1332	0.2190	0.3012	0.3222	2	0.0649	0.3673	0.1704	0.2708	0.3265	0.3522
		300	1	0.0073	0.2257	0.0846	0.1515	0.2514	0.2634	2	0.0224	0.2726	0.1034	0.1810	0.2648	0.2799
		500	1	0.0007	0.1816	0.0619	0.1167	0.2201	0.2284	2	0.0079	0.2264	0.0695	0.1384	0.2296	0.2402
		1000	1	-0.0059	0.1432	0.0377	0.0853	0.1828	0.1881	2	0.0030	0.1759	0.0443	0.0989	0.1894	0.1966
	1.1	150	1	0.0359	0.2922	0.1421	0.2242	0.3047	0.3248	2	0.0506	0.3655	0.1539	0.2617	0.3203	0.3466
		300	1	0.0029	0.2158	0.0860	0.1504	0.2499	0.2622	2	0.0174	0.2699	0.0975	0.1768	0.2655	0.2812
		500	1	-0.0024	0.1804	0.0598	0.1162	0.2207	0.2290	2	0.0049	0.2254	0.0648	0.1344	0.2301	0.2409
		1000	1	-0.0035	0.1458	0.0367	0.0835	0.1822	0.1877	2	-0.0038	0.1715	0.0415	0.0979	0.1873	0.1940
	1.3	150	1	0.0391	0.2914	0.1437	0.2237	0.3073	0.3273	2	0.0405	0.3670	0.1493	0.2582	0.3187	0.3466
		300	1	0.0200	0.2182	0.0939	0.1554	0.2523	0.2641	2	0.0115	0.2722	0.0926	0.1754	0.2666	0.2824
		500	1	0.0076	0.1827	0.0617	0.1167	0.2208	0.2294	2	-0.0018	0.2223	0.0677	0.1366	0.2299	0.2409
		1000	1	-0.0035	0.1445	0.0389	0.0846	0.1838	0.1888	2	-0.0078	0.1762	0.0419	0.0983	0.1875	0.1944
	1.5	150	2	0.0475	0.3605	0.1526	0.2613	0.3200	0.3469	3	0.0995	0.4320	0.1920	0.3128	0.3514	0.3825
		300	2	0.0142	0.2716	0.0939	0.1762	0.2630	0.2789	3	0.0559	0.3234	0.1163	0.2061	0.2792	0.2977
		500	2	-0.0020	0.2270	0.0620	0.1337	0.2271	0.2383	3	0.0289	0.2646	0.0821	0.1589	0.2399	0.2531
		1000	2	-0.0079	0.1760	0.0394	0.0983	0.1866	0.1935	3	0.0171	0.2012	0.0520	0.1102	0.1940	0.2021
	1.7	150	2	0.0372	0.3615	0.1522	0.2593	0.3181	0.3452	3	0.0904	0.4405	0.1850	0.3072	0.3436	0.3768
		300	2	0.0054	0.2732	0.0916	0.1766	0.2631	0.2785	3	0.0468	0.3189	0.1109	0.2056	0.2746	0.2936
		500	2	-0.0023	0.2271	0.0647	0.1360	0.2285	0.2394	3	0.0127	0.2575	0.0797	0.1567	0.2391	0.2522
		1000	2	-0.0074	0.1709	0.0391	0.0967	0.1865	0.1935	3	0.0090	0.2051	0.0484	0.1103	0.1944	0.2028
	1.9	150	2	0.0505	0.3619	0.1505	0.2557	0.3222	0.3501	3	0.0771	0.4390	0.1808	0.3022	0.3431	0.3749
		300	2	0.0071	0.2746	0.0972	0.1778	0.2615	0.2776	3	0.0235	0.3176	0.1051	0.2021	0.2730	0.2923
		500	2	-0.0018	0.2252	0.0668	0.1384	0.2285	0.2396	3	0.0155	0.2553	0.0733	0.1560	0.2379	0.2508
		1000	2	-0.0080	0.1747	0.0376	0.0965	0.1876	0.1946	3	0.0010	0.1995	0.0467	0.1096	0.1931	0.2016

[a] The data generation is $(1 + .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The δ, the difference order, in the expression $d - \delta \in [-.5, .5)/d - \delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d - \delta < .5/-1.5 \le d - \delta < -.5$;
[c] The m is the number of the frequency ordinates involved in the estimation;
[d] The τ is the tapering order.

Table 17: Monte Carlo Results of the Tapered Local Whittle Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .4$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	1	-0.0460	0.2940	-0.0619	0.1786	-0.1898	0.2184	2	0.0077	0.3609	-0.0421	0.2108	-0.1891	0.2278
		300	1	-0.0247	0.2245	-0.0401	0.1300	-0.1242	0.1450	2	0.0003	0.2673	-0.0197	0.1481	-0.1235	0.1520
		500	1	-0.0198	0.1833	-0.0224	0.1018	-0.0967	0.1136	2	0.0069	0.2235	-0.0133	0.1176	-0.0924	0.1148
		1000	1	-0.0159	0.1445	-0.0150	0.0764	-0.0680	0.0803	2	0.0051	0.1744	-0.0050	0.0865	-0.0647	0.0817
	.7	150	1	-0.0415	0.2912	-0.0619	0.1814	-0.1873	0.2164	2	-0.0067	0.3566	-0.0469	0.2126	-0.1960	0.2344
		300	1	-0.0261	0.2232	-0.0356	0.1279	-0.1242	0.1456	2	-0.0074	0.2745	-0.0248	0.1504	-0.1267	0.1551
		500	1	-0.0208	0.1847	-0.0281	0.1026	-0.0957	0.1122	2	-0.0038	0.2240	-0.0178	0.1174	-0.0965	0.1179
		1000	1	-0.0195	0.1463	-0.0153	0.0759	-0.0677	0.0803	2	0.0001	0.1740	-0.0100	0.0880	-0.0675	0.0836
	.9	150	1	-0.0440	0.2929	-0.0631	0.1790	-0.1869	0.2150	2	-0.0192	0.3580	-0.0543	0.2123	-0.1944	0.2317
		300	1	-0.0293	0.2230	-0.0354	0.1286	-0.1229	0.1440	2	-0.0134	0.2644	-0.0300	0.1494	-0.1304	0.1581
		500	1	-0.0243	0.1853	-0.0259	0.1028	-0.0958	0.1118	2	-0.0117	0.2207	-0.0188	0.1184	-0.0971	0.1186
		1000	1	-0.0126	0.1450	-0.0147	0.0751	-0.0677	0.0801	2	-0.0048	0.1762	-0.0104	0.0878	-0.0698	0.0858
	1.1	150	1	-0.0303	0.2943	-0.0601	0.1831	-0.1873	0.2153	2	-0.0349	0.3604	-0.0565	0.2131	-0.2043	0.2408
		300	1	-0.0232	0.2203	-0.0322	0.1266	-0.1233	0.1440	2	-0.0232	0.2754	-0.0337	0.1506	-0.1306	0.1577
		500	1	-0.0190	0.1829	-0.0242	0.1014	-0.0953	0.1114	2	-0.0136	0.2251	-0.0190	0.1170	-0.1003	0.1217
		1000	1	-0.0136	0.1447	-0.0129	0.0738	-0.0670	0.0792	2	-0.0112	0.1769	-0.0135	0.0878	-0.0699	0.0853
	1.3	150	1	-0.0242	0.2862	-0.0527	0.1778	-0.1832	0.2106	2	-0.0394	0.3719	-0.0654	0.2149	-0.2047	0.2402
		300	1	-0.0151	0.2200	-0.0340	0.1265	-0.1212	0.1425	2	-0.0275	0.2722	-0.0362	0.1523	-0.1312	0.1583
		500	1	-0.0086	0.1811	-0.0196	0.0998	-0.0946	0.1113	2	-0.0169	0.2253	-0.0254	0.1187	-0.1015	0.1228
		1000	1	-0.0114	0.1444	-0.0087	0.0752	-0.0678	0.0798	2	-0.0167	0.1758	-0.0151	0.0887	-0.0703	0.0859
	1.5	150	2	-0.0402	0.3573	-0.0673	0.2190	-0.2059	0.2432	3	0.0026	0.4197	-0.0447	0.2390	-0.2057	0.2506
		300	2	-0.0249	0.2715	-0.0383	0.1520	-0.1328	0.1591	3	0.0094	0.3152	-0.0241	0.1705	-0.1317	0.1642
		500	2	-0.0235	0.2278	-0.0272	0.1202	-0.1021	0.1229	3	0.0089	0.2614	-0.0145	0.1339	-0.0990	0.1243
		1000	2	-0.0175	0.1765	-0.0160	0.0888	-0.0708	0.0870	3	0.0049	0.2011	-0.0077	0.0985	-0.0669	0.0864
	1.7	150	2	-0.0507	0.3608	-0.0740	0.2187	-0.2063	0.2420	3	-0.0204	0.4144	-0.0538	0.2443	-0.2105	0.2552
		300	2	-0.0338	0.2743	-0.0392	0.1514	-0.1358	0.1626	3	-0.0113	0.3114	-0.0264	0.1710	-0.1350	0.1682
		500	2	-0.0239	0.2279	-0.0285	0.1213	-0.1025	0.1230	3	-0.0002	0.2570	-0.0170	0.1362	-0.1017	0.1266
		1000	2	-0.0170	0.1714	-0.0165	0.0910	-0.0711	0.0863	3	-0.0004	0.2028	-0.0079	0.0976	-0.0703	0.0895
	1.9	150	2	-0.0377	0.3581	-0.0691	0.2141	-0.2060	0.2416	3	-0.0196	0.4193	-0.0644	0.2398	-0.2180	0.2617
		300	2	-0.0321	0.2754	-0.0380	0.1525	-0.1358	0.1626	3	-0.0145	0.3194	-0.0359	0.1739	-0.1366	0.1691
		500	2	-0.0233	0.2260	-0.0272	0.1221	-0.1012	0.1223	3	-0.0150	0.2633	-0.0225	0.1336	-0.1023	0.1273
		1000	2	-0.0176	0.1752	-0.0155	0.0886	-0.0707	0.0860	3	-0.0120	0.2005	-0.0118	0.0980	-0.0718	0.0898

[a] The data generation is $(1-.4L)(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;

[b] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;

[c] The m is the number of the frequency ordinates involved in the estimation;

[d] The τ is the tapering order.

Table 18: Monte Carlo Results of the Tapered Local Whittle Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .8$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
ϕ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	1	-0.0387	0.2854	-0.0684	0.1838	-0.2639	0.2850	2	0.0048	0.3558	-0.0553	0.2127	-0.2710	0.3007
		300	1	-0.0267	0.2239	-0.0406	0.1315	-0.1641	0.1817	2	0.0066	0.2656	-0.0269	0.1514	-0.1633	0.1860
		500	1	-0.0214	0.1843	-0.0286	0.1033	-0.1236	0.1368	2	0.0050	0.2220	-0.0147	0.1181	-0.1227	0.1402
		1000	1	-0.0156	0.1457	-0.0155	0.0770	-0.0859	0.0956	2	0.0090	0.1712	-0.0107	0.0888	-0.0830	0.0973
	.7	150	1	-0.0476	0.2949	-0.0687	0.1822	-0.2626	0.2836	2	-0.0038	0.3583	-0.0611	0.2159	-0.2778	0.3055
		300	1	-0.0294	0.2239	-0.0424	0.1321	-0.1646	0.1812	2	-0.0056	0.2708	-0.0341	0.1517	-0.1672	0.1892
		500	1	-0.0223	0.1848	-0.0290	0.1025	-0.1210	0.1348	2	-0.0036	0.2249	-0.0200	0.1196	-0.1250	0.1430
		1000	1	-0.0140	0.1440	-0.0172	0.0758	-0.0849	0.0949	2	-0.0048	0.1723	-0.0098	0.0880	-0.0854	0.0986
	.9	150	1	-0.0404	0.2918	-0.0732	0.1839	-0.2642	0.2858	2	-0.0283	0.3558	-0.0629	0.2160	-0.2839	0.3121
		300	1	-0.0290	0.2225	-0.0440	0.1292	-0.1637	0.1810	2	-0.0125	0.2712	-0.0339	0.1528	-0.1717	0.1930
		500	1	-0.0224	0.1840	-0.0295	0.1031	-0.1260	0.1393	2	-0.0086	0.2181	-0.0250	0.1222	-0.1268	0.1440
		1000	1	-0.0172	0.1450	-0.0184	0.0765	-0.0854	0.0957	2	-0.0040	0.1762	-0.0137	0.0894	-0.0864	0.0998
	1.1	150	1	-0.0407	0.2877	-0.0685	0.1857	-0.2618	0.2828	2	-0.0322	0.3501	-0.0720	0.2183	-0.2855	0.3134
		300	1	-0.0240	0.2236	-0.0384	0.1289	-0.1618	0.1785	2	-0.0212	0.2712	-0.0427	0.1536	-0.1732	0.1946
		500	1	-0.0201	0.1830	-0.0299	0.1032	-0.1241	0.1373	2	-0.0182	0.2230	-0.0288	0.1214	-0.1299	0.1466
		1000	1	-0.0114	0.1402	-0.0159	0.0766	-0.0845	0.0946	2	-0.0149	0.1767	-0.0138	0.0900	-0.0873	0.1001
	1.3	150	1	-0.0202	0.2833	-0.0676	0.1831	-0.2579	0.2794	2	0.0521	0.3676	-0.0727	0.2173	-0.2890	0.3163
		300	1	-0.0227	0.2206	-0.0345	0.1278	-0.1635	0.1797	2	-0.0339	0.2714	-0.0430	0.1530	-0.1760	0.1970
		500	1	-0.0115	0.1800	-0.0255	0.1010	-0.1208	0.1342	2	-0.0203	0.2252	-0.0264	0.1210	-0.1289	0.1459
		1000	1	-0.0106	0.1449	-0.0140	0.0741	-0.0840	0.0943	2	-0.0168	0.1735	-0.0198	0.0915	-0.0887	0.1016
	1.5	150	2	-0.0466	0.3569	-0.0748	0.2194	-0.2916	0.3185	3	-0.0014	0.4185	-0.0600	0.2460	-0.2974	0.3329
		300	2	-0.0330	0.2748	-0.0449	0.1548	-0.1753	0.1968	3	0.0011	0.3117	-0.0337	0.1737	-0.1750	0.2016
		500	2	-0.0256	0.2247	-0.0324	0.1229	-0.1306	0.1477	3	0.0034	0.2559	-0.0198	0.1364	-0.1279	0.1486
		1000	2	-0.0116	0.1722	-0.0186	0.0910	-0.0889	0.1018	3	0.0022	0.2011	-0.0114	0.1021	-0.0872	0.1031
	1.7	150	2	-0.0598	0.3628	-0.0809	0.2238	-0.2937	0.3207	3	-0.0197	0.4304	-0.0651	0.2453	-0.3084	0.3416
		300	2	-0.0356	0.2694	-0.0455	0.1557	-0.1773	0.1977	3	-0.0122	0.3158	-0.0361	0.1723	-0.1807	0.2060
		500	2	-0.0265	0.2215	-0.0315	0.1212	-0.1311	0.1479	3	-0.0080	0.2580	-0.0251	0.1345	-0.1315	0.1519
		1000	2	-0.0175	0.1779	-0.0194	0.0903	-0.0902	0.1028	3	-0.0028	0.2005	-0.0125	0.1010	-0.0887	0.1041
	1.9	150	2	-0.0439	0.3564	-0.0795	0.2197	-0.2910	0.3182	3	-0.0306	0.4247	-0.0760	0.2511	-0.3129	0.3461
		300	2	-0.0304	0.2734	-0.0437	0.1556	-0.1754	0.1960	3	-0.0132	0.3175	-0.0402	0.1744	-0.1823	0.2080
		500	2	-0.0250	0.2261	-0.0300	0.1192	-0.1312	0.1475	3	-0.0125	0.2641	-0.0260	0.1356	-0.1335	0.1539
		1000	2	-0.0154	0.1761	-0.0189	0.0902	-0.0898	0.1024	3	-0.0080	0.2027	-0.0162	0.0992	-0.0900	0.1056

[a] The data generation is $(1-.8L)(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[c] The m is the number of the frequency ordinates involved in the estimation;
[d] The τ is the tapering order.

Table 19: Monte Carlo Results of the Tapered Local Whittle Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	1	-0.0987	0.3040	-0.1669	0.2378	-0.3018	0.3207	2	-0.0490	0.3561	-0.1569	0.2579	-0.3049	0.3306
		300	1	-0.0494	0.2278	-0.1082	0.1650	-0.2389	0.2507	2	-0.0214	0.2715	-0.0972	0.1732	-0.2430	0.2588
		500	1	-0.0290	0.1830	-0.0767	0.1250	-0.2076	0.2163	2	-0.0063	0.2231	-0.0680	0.1352	-0.2102	0.2213
		1000	1	-0.0218	0.1483	-0.0487	0.0886	-0.1708	0.1763	2	-0.0001	0.1741	-0.0433	0.0972	-0.1715	0.1789
	.7	150	1	-0.1011	0.3037	-0.1676	0.2388	-0.3010	0.3199	2	-0.0790	0.3657	-0.1684	0.2662	-0.3122	0.3375
		300	1	-0.0470	0.2292	-0.1097	0.1643	-0.2416	0.2536	2	-0.0279	0.2721	-0.1068	0.1798	-0.2489	0.2645
		500	1	-0.0376	0.1841	-0.0760	0.1249	-0.2085	0.2171	2	-0.0197	0.2200	-0.0742	0.1381	-0.2137	0.2246
		1000	1	-0.0210	0.1460	-0.0485	0.0881	-0.1696	0.1751	2	-0.0089	0.1731	-0.0455	0.0983	-0.1724	0.1797
	.9	150	1	-0.0871	0.3040	-0.1674	0.2384	-0.2998	0.3183	2	-0.0837	0.3673	-0.1691	0.2648	-0.3144	0.3387
		300	1	-0.0506	0.2259	-0.1050	0.1617	-0.2413	0.2529	2	-0.0466	0.2714	-0.1135	0.1863	-0.2497	0.2646
		500	1	-0.0329	0.1807	-0.0762	0.1248	-0.2080	0.2164	2	-0.0260	0.2223	-0.0766	0.1407	-0.2159	0.2270
		1000	1	-0.0209	0.1453	-0.0489	0.0892	-0.1706	0.1762	2	-0.0189	0.1745	-0.0465	0.0995	-0.1738	0.1810
	1.1	150	1	-0.0810	0.2944	-0.1592	0.2334	-0.2980	0.3171	2	-0.0822	0.3662	-0.1781	0.2718	-0.3204	0.3444
		300	1	-0.0508	0.2262	-0.1057	0.1615	-0.2419	0.2539	2	-0.0473	0.2732	-0.1164	0.1861	-0.2547	0.2705
		500	1	-0.0290	0.1846	-0.0746	0.1227	-0.2071	0.2153	2	-0.0345	0.2288	-0.0781	0.1408	-0.2169	0.2276
		1000	1	-0.0180	0.1444	-0.0462	0.0867	-0.1708	0.1763	2	-0.0160	0.1727	-0.0507	0.1005	-0.1760	0.1830
	1.3	150	1	-0.0693	0.2874	-0.1596	0.2295	-0.2941	0.3131	2	-0.0956	0.3719	-0.1826	0.2778	-0.3235	0.3481
		300	1	-0.0395	0.2222	-0.1017	0.1582	-0.2386	0.2507	2	-0.0512	0.2743	-0.1178	0.1888	-0.2557	0.2712
		500	1	-0.0265	0.1800	-0.0706	0.1210	-0.2060	0.2141	2	-0.0322	0.2255	-0.0827	0.1436	-0.2169	0.2275
		1000	1	-0.0105	0.1436	-0.0466	0.0882	-0.1683	0.1739	2	-0.0223	0.1755	-0.0507	0.1013	-0.1771	0.1842
	1.5	150	2	-0.1074	0.3665	-0.1807	0.2732	-0.3251	0.3487	3	-0.0555	0.4233	-0.1766	0.2955	-0.3250	0.3546
		300	2	-0.0561	0.2784	-0.1154	0.1881	-0.2584	0.2737	3	-0.0295	0.3133	-0.1140	0.2019	-0.2610	0.2796
		500	2	-0.0299	0.2246	-0.0847	0.1455	-0.2197	0.2305	3	-0.0116	0.2588	-0.0748	0.1509	-0.2200	0.2330
		1000	2	-0.0227	0.1755	-0.0517	0.1008	-0.1781	0.1850	3	-0.0041	0.1997	-0.0443	0.1082	-0.1780	0.1862
	1.7	150	2	-0.1077	0.3730	-0.1872	0.2755	-0.3228	0.3468	3	-0.0933	0.4409	-0.1849	0.3017	-0.3322	0.3627
		300	2	-0.0489	0.2752	-0.1190	0.1892	-0.2560	0.2708	3	-0.0394	0.3182	-0.1190	0.2038	-0.2636	0.2823
		500	2	-0.0384	0.2297	-0.0834	0.1437	-0.2184	0.2293	3	-0.0241	0.2604	-0.0784	0.1537	-0.2234	0.2364
		1000	2	-0.0223	0.1760	-0.0544	0.1041	-0.1781	0.1850	3	-0.0078	0.2002	-0.0488	0.1114	-0.1790	0.1870
	1.9	150	2	-0.0971	0.3715	-0.1842	0.2741	-0.3258	0.3496	3	-0.0881	0.4291	-0.1941	0.3062	-0.3353	0.3654
		300	2	-0.0514	0.2796	-0.1195	0.1886	-0.2580	0.2729	3	-0.0465	0.3230	-0.1152	0.2017	-0.2638	0.2818
		500	2	-0.0344	0.2263	-0.0842	0.1446	-0.2217	0.2320	3	-0.0288	0.2607	-0.0827	0.1564	-0.2240	0.2371
		1000	2	-0.0255	0.1774	-0.0525	0.1020	-0.1781	0.1848	3	-0.0189	0.1994	-0.0531	0.1117	-0.1809	0.1894

[a] The data generation is $(1-L)^d X_t = (1-.4L)\varepsilon_t$, ,$\varepsilon_t \sim N(0,1)$;
[b] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[c] The m is the number of the frequency ordinates involved in the estimation;
[d] The τ is the tapering order.

Table 20: Monte Carlo Results of the Tapered Local Whittle Estimation for ARFIMA$(0,d,1)$ Model with $\theta = .4$

				$d-\delta \in [-.5,.5)$							$d-\delta \in [-1.5,-.5)$					
				$m=\lfloor n^{.5} \rfloor$		$m=\lfloor n^{.65} \rfloor$		$m=\lfloor n^{.8} \rfloor$			$m=\lfloor n^{.5} \rfloor$		$m=\lfloor n^{.65} \rfloor$		$m=\lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	1	-0.0148	0.2876	0.0271	0.1718	0.1687	0.2025	2	0.0397	0.3546	0.0584	0.2162	0.1986	0.2380
		300	1	-0.0196	0.2230	0.0103	0.1225	0.1143	0.1380	2	0.0142	0.2708	0.0297	0.1479	0.1297	0.1591
		500	1	-0.0139	0.1866	0.0043	0.0988	0.0888	0.1069	2	0.0134	0.2228	0.0211	0.1179	0.1002	0.1219
		1000	1	-0.0144	0.1474	0.0052	0.0740	0.0635	0.0770	2	0.0095	0.1709	0.0123	0.0886	0.0707	0.0867
	.7	150	1	-0.0178	0.2839	0.0267	0.1749	0.1661	0.1991	2	0.0188	0.3568	0.0510	0.2148	0.1913	0.2322
		300	1	-0.0213	0.2189	0.0094	0.1240	0.1138	0.1372	2	0.0078	0.2732	0.0300	0.1501	0.1265	0.1554
		500	1	-0.0183	0.1849	0.0046	0.0998	0.0888	0.1069	2	0.0072	0.2246	0.0186	0.1206	0.0965	0.1189
		1000	1	-0.0096	0.1431	0.0023	0.0728	0.0636	0.0770	2	0.0055	0.1732	0.0081	0.0880	0.0682	0.0841
	.9	150	1	-0.0119	0.2931	0.0254	0.1705	0.1677	0.2010	2	0.0059	0.3661	0.0436	0.2120	0.1894	0.2306
		300	1	-0.0185	0.2240	0.0145	0.1221	0.1139	0.1379	2	-0.0034	0.2706	0.0210	0.1482	0.1274	0.1560
		500	1	-0.0154	0.1822	0.0059	0.0998	0.0894	0.1079	2	-0.0025	0.2227	0.0114	0.1182	0.0949	0.1178
		1000	1	-0.0155	0.1448	0.0046	0.0747	0.0638	0.0772	2	-0.0029	0.1779	0.0059	0.0886	0.0679	0.0847
	1.1	150	1	-0.0123	0.2919	0.0243	0.1706	0.1706	0.2031	2	-0.0010	0.3565	0.0332	0.2121	0.1865	0.2284
		300	1	-0.0151	0.2237	0.0176	0.1247	0.1136	0.1377	2	-0.0048	0.2692	0.0188	0.1471	0.1227	0.1531
		500	1	-0.0133	0.1833	0.0069	0.1007	0.0901	0.1076	2	-0.0158	0.2310	0.0089	0.1187	0.0956	0.1183
		1000	1	-0.0127	0.1438	0.0047	0.0744	0.0641	0.0771	2	-0.0043	0.1748	0.0048	0.0875	0.0671	0.0834
	1.3	150	1	0.0019	0.2877	0.0324	0.1712	0.1732	0.2043	2	-0.0147	0.3602	0.0295	0.2080	0.1858	0.2280
		300	1	-0.0139	0.2112	0.0177	0.1243	0.1157	0.1387	2	-0.0157	0.2724	0.0159	0.1495	0.1216	0.1515
		500	1	-0.0084	0.1816	0.0094	0.1000	0.0899	0.1077	2	-0.0144	0.2244	0.0097	0.1195	0.0942	0.1172
		1000	1	-0.0037	0.1422	0.0065	0.0749	0.0654	0.0786	2	-0.0098	0.1737	0.0017	0.0868	0.0672	0.0839
	1.5	150	2	-0.0157	0.3590	0.0345	0.2113	0.1844	0.2276	3	0.0326	0.4161	0.0606	0.2531	0.2116	0.2589
		300	2	-0.0206	0.2741	0.0134	0.1474	0.1220	0.1519	3	0.0223	0.3176	0.0276	0.1725	0.1343	0.1677
		500	2	-0.0109	0.2270	0.0059	0.1171	0.0923	0.1146	3	0.0131	0.2561	0.0187	0.1356	0.1026	0.1286
		1000	2	-0.0157	0.1774	0.0025	0.0864	0.0660	0.0824	3	0.0080	0.1992	0.0127	0.0988	0.0727	0.0914
	1.7	150	2	-0.0213	0.3653	0.0278	0.2060	0.1833	0.2274	3	0.0275	0.4202	0.0456	0.2456	0.2044	0.2527
		300	2	-0.0220	0.2715	0.0136	0.1495	0.1199	0.1507	3	-0.0011	0.3156	0.0281	0.1726	0.1310	0.1665
		500	2	-0.0164	0.2247	0.0085	0.1218	0.0930	0.1165	3	0.0043	0.2643	0.0157	0.1342	0.1010	0.1270
		1000	2	-0.0151	0.1756	0.0022	0.0884	0.0650	0.0815	3	0.0017	0.2031	0.0097	0.0997	0.0700	0.0886
	1.9	150	2	-0.0188	0.3556	0.0320	0.2080	0.1807	0.2235	3	0.0205	0.4300	0.0457	0.2484	0.2058	0.2563
		300	2	-0.0165	0.2717	0.0128	0.1498	0.1217	0.1529	3	-0.0020	0.3205	0.0190	0.1720	0.1297	0.1648
		500	2	-0.0187	0.2253	0.0055	0.1191	0.0925	0.1158	3	-0.0022	0.2575	0.0127	0.1379	0.1006	0.1281
		1000	2	-0.0149	0.1758	0.0043	0.0889	0.0669	0.0832	3	-0.0032	0.1987	0.0084	0.0988	0.0699	0.0888

[a] The data generation is $(1-L)^d X_t = (1+.4L)\varepsilon_t$, $,\varepsilon_t \sim N(0,1)$;
[b] The δ, the difference order, in the expression $d-\delta \in [-.5,.5)/d-\delta \in [-1.5,-.5)$ is such that for the true value d, select a δ satisfying $-.5 \le d-\delta < .5/-1.5 \le d-\delta < -.5$;
[c] The m is the number of the frequency ordinates involved in the estimation;
[d] The τ is the tapering order.

Table 21: Monte Carlo Results of the Tapered Local Whittle Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$

				$d-\delta \in [-.5, .5)$							$d-\delta \in [-1.5, -.5)$					
				$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$			$m = \lfloor n^{.5} \rfloor$		$m = \lfloor n^{.65} \rfloor$		$m = \lfloor n^{.8} \rfloor$	
θ	d	n	τ	bias	RMSE	bias	RMSE	bias	RMSE	τ	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	1	-0.0132	0.2910	0.0331	0.1751	0.2381	0.2647	2	0.0410	0.3591	0.0646	0.2196	0.2349	0.2615
		300	1	-0.0146	0.2211	0.0226	0.1241	0.1526	0.1713	2	0.0220	0.2704	0.0360	0.1528	0.1543	0.1732
		500	1	-0.0174	0.1863	0.0096	0.1002	0.1168	0.1312	2	0.0117	0.2182	0.0236	0.1187	0.1173	0.1320
		1000	1	-0.0131	0.1451	0.0069	0.0748	0.0818	0.0928	2	0.0091	0.1742	0.0151	0.0889	0.0820	0.0928
	.7	150	1	-0.0142	0.2833	0.0359	0.1733	0.2363	0.2627	2	0.0183	0.3595	0.0637	0.2174	0.2396	0.2650
		300	1	-0.0163	0.2234	0.0136	0.1249	0.1539	0.1727	2	0.0042	0.2710	0.0354	0.1503	0.1517	0.1700
		500	1	-0.0106	0.1812	0.0091	0.1009	0.1167	0.1312	2	0.0027	0.2202	0.0197	0.1184	0.1172	0.1319
		1000	1	-0.0116	0.1448	0.0046	0.0742	0.0814	0.0919	2	0.0026	0.1716	0.0105	0.0889	0.0826	0.0934
	.9	150	1	-0.0152	0.2886	0.0379	0.1735	0.2380	0.2639	2	0.0109	0.3615	0.0524	0.2147	0.2380	0.2642
		300	1	-0.0121	0.2189	0.0169	0.1233	0.1534	0.1723	2	-0.0034	0.2751	0.0258	0.1498	0.1532	0.1725
		500	1	-0.0200	0.1853	0.0104	0.1001	0.1173	0.1320	2	-0.0038	0.2222	0.0172	0.1192	0.1174	0.1315
		1000	1	-0.0130	0.1471	0.0043	0.0756	0.0821	0.0925	2	-0.0064	0.1738	0.0072	0.0878	0.0818	0.0924
	1.1	150	1	-0.0067	0.2922	0.0385	0.1730	0.2377	0.2628	2	-0.0014	0.3642	0.0454	0.2090	0.2378	0.2638
		300	1	-0.0149	0.2198	0.0184	0.1234	0.1522	0.1705	2	-0.0073	0.2726	0.0179	0.1524	0.1515	0.1706
		500	1	-0.0141	0.1841	0.0113	0.1008	0.1176	0.1320	2	-0.0093	0.2266	0.0140	0.1187	0.1181	0.1322
		1000	1	-0.0106	0.1445	0.0057	0.0734	0.0827	0.0932	2	-0.0063	0.1789	0.0078	0.0876	0.0828	0.0929
	1.3	150	1	-0.0014	0.2875	0.0443	0.1748	0.2397	0.2645	2	0.0014	0.3598	0.0414	0.2111	0.2397	0.2649
		300	1	-0.0099	0.2190	0.0258	0.1246	0.1554	0.1738	2	-0.0145	0.2704	0.0228	0.1501	0.1549	0.1728
		500	1	-0.0056	0.1814	0.0130	0.1002	0.1179	0.1327	2	-0.0107	0.2196	0.0115	0.1182	0.1176	0.1319
		1000	1	-0.0044	0.1427	0.0076	0.0743	0.0824	0.0928	2	-0.0081	0.1764	0.0062	0.0885	0.0818	0.0927
	1.5	150	2	-0.0137	0.3586	0.0421	0.2143	0.2621	0.2955	3	0.0441	0.4246	0.0742	0.2553	0.2605	0.2945
		300	2	-0.0139	0.2696	0.0200	0.1516	0.1622	0.1866	3	0.0183	0.3152	0.0405	0.1733	0.1618	0.1862
		500	2	-0.0168	0.2280	0.0090	0.1184	0.1222	0.1412	3	0.0042	0.2554	0.0282	0.1377	0.1213	0.1407
		1000	2	-0.0147	0.1770	0.0050	0.0891	0.0850	0.0988	3	0.0068	0.1947	0.0144	0.0998	0.0846	0.0984
	1.7	150	2	-0.0143	0.3639	0.0391	0.2105	0.2565	0.2919	3	0.0332	0.4208	0.0665	0.2522	0.2568	0.2907
		300	2	-0.0113	0.2720	0.0187	0.1536	0.1627	0.1871	3	0.0074	0.3164	0.0334	0.1704	0.1598	0.1844
		500	2	-0.0172	0.2254	0.0091	0.1185	0.1212	0.1402	3	0.0088	0.2542	0.0228	0.1341	0.1218	0.1409
		1000	2	-0.0138	0.1718	0.0036	0.0878	0.0835	0.0978	3	0.0067	0.1983	0.0112	0.0997	0.0842	0.0975
	1.9	150	2	-0.0055	0.3528	0.0359	0.2089	0.2580	0.2910	3	0.0191	0.4329	0.0601	0.2484	0.2630	0.2972
		300	2	-0.0133	0.2642	0.0217	0.1498	0.1630	0.1872	3	-0.0001	0.3170	0.0289	0.1729	0.1621	0.1864
		500	2	-0.0176	0.2268	0.0074	0.1189	0.1232	0.1423	3	-0.0021	0.2584	0.0189	0.1365	0.1211	0.1394
		1000	2	-0.0134	0.1742	0.0033	0.0874	0.0858	0.0993	3	-0.0040	0.2024	0.0085	0.0999	0.0843	0.0987

[a] The data generation is $(1-L)^d X_t = (1+.8L)\varepsilon_t$, ,$\varepsilon_t \sim N(0,1)$;
[b] The δ, the difference order, in the expression $d-\delta \in [-.5, .5)/d-\delta \in [-1.5, -.5)$ is such that for the true value d, select a δ satisfying $-.5 \leq d-\delta < .5/-1.5 \leq d-\delta < -.5$;
[c] The m is the number of the frequency ordinates involved in the estimation;
[d] The τ is the tapering order.

Table 22: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 0)$ Model with $m = \lfloor n^{.5} \rfloor$

		fextLWF		fextLPWF		ModLWF		feLWF		dtr-feLWF	
d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	0.0196	0.2188	-0.1419	0.4167	0.0213	0.2183	0.0097	0.2073	-0.0877	0.2560
	300	0.0099	0.1748	-0.0887	0.3012	0.0254	0.1745	0.0055	0.1675	-0.0661	0.2027
	500	0.0155	0.1496	-0.0612	0.2395	0.0264	0.1526	0.0084	0.1442	-0.0473	0.1669
	1000	0.0115	0.1185	-0.0481	0.1809	0.0297	0.1234	0.0080	0.1184	-0.0318	0.1320
.7	150	-0.0078	0.2064	-0.1389	0.4265	-0.0134	0.2181	0.0027	0.1963	-0.0759	0.2611
	300	-0.0034	0.1627	-0.0988	0.3148	-0.0108	0.1703	0.0003	0.1544	-0.0603	0.2005
	500	-0.0086	0.1430	-0.0728	0.2554	-0.0019	0.1408	0.0005	0.1236	-0.0445	0.1604
	1000	-0.0018	0.1154	-0.0448	0.1961	-0.0005	0.1125	-0.0022	0.1025	-0.0334	0.1251
.9	150	-0.0276	0.2147	-0.1417	0.4336	-0.0288	0.2216	-0.0138	0.2034	-0.0652	0.2612
	300	-0.0192	0.1688	-0.0969	0.3329	-0.0227	0.1726	-0.0133	0.1608	-0.0463	0.1954
	500	-0.0164	0.1453	-0.0620	0.2664	-0.0153	0.1437	-0.0148	0.1411	-0.0354	0.1598
	1000	-0.0097	0.1147	-0.0378	0.2020	-0.0100	0.1105	-0.0104	0.1127	-0.0193	0.1258
1.1	150	-0.0288	0.2259	-0.1358	0.4490	-0.0343	0.2236	-0.0193	0.2177	-0.0470	0.2550
	300	-0.0263	0.1752	-0.0823	0.3341	-0.0233	0.1671	-0.0211	0.1742	-0.0224	0.1814
	500	-0.0194	0.1425	-0.0675	0.2733	-0.0129	0.1445	-0.0175	0.1459	-0.0177	0.1466
	1000	-0.0063	0.1111	-0.0337	0.2010	-0.0120	0.1118	-0.0112	0.1131	-0.0135	0.1146
1.3	150	-0.0199	0.2309	-0.1066	0.4408	-0.0303	0.2184	-0.0293	0.2335	-0.0186	0.2292
	300	-0.0158	0.1792	-0.0616	0.3230	-0.0180	0.1725	-0.0203	0.1782	-0.0157	0.1709
	500	-0.0057	0.1498	-0.0381	0.2620	-0.0129	0.1410	-0.0152	0.1481	-0.0105	0.1420
	1000	-0.0052	0.1187	-0.0356	0.1958	-0.0086	0.1146	-0.0106	0.1192	-0.0074	0.1121
1.5	150	-0.0099	0.2211	-0.0889	0.4241	-0.0227	0.2215	-0.0420	0.2511	-0.0093	0.2186
	300	0.0003	0.1755	-0.0563	0.3104	-0.0047	0.1695	-0.0317	0.2036	-0.0060	0.1647
	500	0.0055	0.1488	-0.0408	0.2503	-0.0004	0.1451	-0.0265	0.1687	-0.0027	0.1388
	1000	0.0123	0.1202	-0.0437	0.1864	0.0027	0.1158	-0.0115	0.1294	-0.0020	0.1101
1.7	150	-0.0238	0.2119	-0.0820	0.4154	-0.0114	0.2162	-0.0293	0.2348	0.0021	0.2091
	300	-0.0121	0.1669	-0.0496	0.2998	0.0032	0.1727	-0.0245	0.1827	0.0019	0.1661
	500	-0.0064	0.1403	-0.0367	0.2397	0.0072	0.1432	-0.0164	0.1471	0.0054	0.1386
	1000	-0.0047	0.1118	-0.0781	0.2017	0.0152	0.1159	-0.0100	0.1162	0.0042	0.1107
1.9	150	-0.0330	0.2143	-0.0757	0.4113	-0.0333	0.2104	-0.0274	0.2315	0.0085	0.2045
	300	-0.0171	0.1676	-0.0712	0.3087	-0.0104	0.1607	-0.0255	0.1750	0.0050	0.1587
	500	-0.0142	0.1416	-0.0551	0.2518	0.0006	0.1356	-0.0194	0.1431	0.0087	0.1341
	1000	-0.0104	0.1142	-0.0574	0.2168	0.0065	0.1091	-0.0116	0.1109	0.0065	0.1103

[a] The data generation is $(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 23: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 0)$ Model with $m = \lfloor n^{.65} \rfloor$

		fextLWF		fextLPWF		ModLWF		feLWF		dtr-feLWF	
d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	-0.0008	0.1318	-0.0477	0.2229	0.0115	0.1342	0.0194	0.1367	-0.0290	0.1516
	300	0.0005	0.0978	-0.0230	0.1502	0.0137	0.1015	0.0168	0.1024	-0.0138	0.1118
	500	0.0012	0.0793	-0.0190	0.1249	0.0138	0.0840	0.0150	0.0869	-0.0126	0.0898
	1000	0.0018	0.0621	-0.0057	0.0895	0.0133	0.0660	0.0129	0.0691	-0.0059	0.0680
.7	150	-0.0168	0.1318	-0.0566	0.2333	-0.0114	0.1304	0.0106	0.1204	-0.0275	0.1471
	300	-0.0096	0.0989	-0.0375	0.1693	-0.0067	0.0974	0.0029	0.0897	-0.0218	0.1024
	500	-0.0043	0.0798	-0.0270	0.1454	-0.0039	0.0789	0.0012	0.0719	-0.0142	0.0816
	1000	-0.0034	0.0612	-0.0154	0.1057	-0.0002	0.0604	0.0001	0.0558	-0.0073	0.0625
.9	150	-0.0253	0.1334	-0.0649	0.2496	-0.0218	0.1291	-0.0000	0.1322	-0.0163	0.1460
	300	-0.0150	0.0956	-0.0303	0.1712	-0.0135	0.0947	-0.0023	0.0986	-0.0101	0.1055
	500	-0.0131	0.0781	-0.0257	0.1436	-0.0126	0.0796	-0.0030	0.0798	-0.0061	0.0844
	1000	-0.0080	0.0588	-0.0148	0.0973	-0.0058	0.0598	-0.0024	0.0603	-0.0048	0.0604
1.1	150	-0.0272	0.1338	-0.0474	0.2625	-0.0306	0.1344	-0.0027	0.1345	0.0003	0.1323
	300	-0.0185	0.0984	-0.0279	0.1715	-0.0193	0.0953	-0.0015	0.0965	0.0013	0.0944
	500	-0.0147	0.0802	-0.0222	0.1368	-0.0134	0.0799	-0.0036	0.0772	-0.0043	0.0776
	1000	-0.0077	0.0597	-0.0126	0.0986	-0.0085	0.0597	-0.0006	0.0586	-0.0021	0.0593
1.3	150	-0.0275	0.1364	-0.0353	0.2626	-0.0286	0.1328	0.0027	0.1379	0.0059	0.1317
	300	-0.0141	0.0998	-0.0115	0.1732	-0.0161	0.0967	0.0003	0.0980	0.0026	0.0966
	500	-0.0093	0.0797	-0.0042	0.1418	-0.0100	0.0783	0.0011	0.0779	0.0002	0.0772
	1000	-0.0036	0.0608	-0.0018	0.1038	-0.0053	0.0596	0.0013	0.0591	0.0020	0.0594
1.5	150	-0.0164	0.1301	-0.0343	0.2652	-0.0280	0.1345	-0.0043	0.1504	0.0104	0.1272
	300	-0.0062	0.1007	-0.0115	0.1606	-0.0139	0.0985	-0.0013	0.1063	0.0070	0.0933
	500	0.0035	0.0818	-0.0037	0.1273	-0.0079	0.0802	0.0030	0.0848	0.0054	0.0769
	1000	0.0088	0.0634	0.0021	0.0945	-0.0007	0.0602	0.0043	0.0622	0.0055	0.0591
1.7	150	-0.0392	0.1313	-0.0447	0.2575	-0.0383	0.1380	-0.0058	0.1366	0.0162	0.1290
	300	-0.0188	0.0959	-0.0170	0.1584	-0.0118	0.0976	0.0015	0.0975	0.0117	0.0968
	500	-0.0111	0.0776	-0.0132	0.1331	0.0012	0.0794	-0.0005	0.0794	0.0099	0.0787
	1000	-0.0057	0.0596	-0.0079	0.0977	0.0071	0.0634	0.0014	0.0598	0.0080	0.0606
1.9	150	-0.0510	0.1387	-0.0604	0.2767	-0.0746	0.1467	-0.0002	0.1325	0.0167	0.1262
	300	-0.0280	0.1006	-0.0290	0.1643	-0.0385	0.0991	-0.0028	0.0968	0.0133	0.0925
	500	-0.0188	0.0797	-0.0176	0.1317	-0.0205	0.0779	-0.0017	0.0778	0.0122	0.0762
	1000	-0.0113	0.0606	-0.0140	0.0956	-0.0031	0.0570	-0.0009	0.0589	0.0093	0.0594

[a] The data generation is $(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 24: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 0)$ Model with $m = \lfloor n^{.8} \rfloor$

		fextLWF		fextLPWF		ModLWF		feLWF		dtr-feLWF	
d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	-0.0224	0.0839	-0.0064	0.1275	-0.0175	0.0839	0.0785	0.1162	0.0571	0.1098
	300	-0.0158	0.0599	-0.0029	0.0889	-0.0090	0.0610	0.0545	0.0846	0.0371	0.0772
	500	-0.0133	0.0471	0.0046	0.0698	-0.0063	0.0486	0.0414	0.0672	0.0312	0.0618
	1000	-0.0098	0.0353	0.0037	0.0507	-0.0034	0.0368	0.0291	0.0493	0.0210	0.0447
.7	150	-0.0419	0.0903	-0.0139	0.1323	-0.0445	0.0912	0.0634	0.1021	0.0540	0.1039
	300	-0.0277	0.0641	-0.0077	0.0969	-0.0290	0.0638	0.0390	0.0701	0.0330	0.0712
	500	-0.0223	0.0503	-0.0048	0.0750	-0.0227	0.0504	0.0294	0.0545	0.0238	0.0551
	1000	-0.0166	0.0378	-0.0004	0.0544	-0.0168	0.0381	0.0200	0.0397	0.0174	0.0394
.9	150	-0.0594	0.0989	-0.0214	0.1322	-0.0625	0.1009	0.0598	0.1043	0.0568	0.1032
	300	-0.0421	0.0709	-0.0119	0.0950	-0.0432	0.0717	0.0369	0.0708	0.0368	0.0708
	500	-0.0320	0.0547	-0.0085	0.0735	-0.0326	0.0551	0.0288	0.0552	0.0279	0.0551
	1000	-0.0236	0.0412	-0.0031	0.0521	-0.0239	0.0413	0.0193	0.0391	0.0200	0.0399
1.1	150	-0.0706	0.1053	-0.0208	0.1345	-0.0744	0.1083	0.0611	0.1035	0.0605	0.1024
	300	-0.0493	0.0754	-0.0117	0.0942	-0.0508	0.0765	0.0387	0.0712	0.0376	0.0705
	500	-0.0408	0.0601	-0.0092	0.0723	-0.0416	0.0607	0.0280	0.0544	0.0280	0.0551
	1000	-0.0288	0.0443	-0.0037	0.0518	-0.0291	0.0445	0.0194	0.0388	0.0204	0.0401
1.3	150	-0.0798	0.1131	-0.0124	0.1352	-0.0848	0.1170	0.0640	0.1076	0.0629	0.1048
	300	-0.0550	0.0792	-0.0072	0.0961	-0.0573	0.0809	0.0410	0.0730	0.0389	0.0712
	500	-0.0441	0.0630	-0.0023	0.0737	-0.0453	0.0638	0.0289	0.0549	0.0301	0.0553
	1000	-0.0329	0.0474	0.0006	0.0524	-0.0334	0.0478	0.0205	0.0398	0.0207	0.0397
1.5	150	-0.0857	0.1188	-0.0152	0.1211	-0.1036	0.1323	0.0651	0.1100	0.0649	0.1057
	300	-0.0551	0.0821	-0.0063	0.0845	-0.0672	0.0894	0.0422	0.0750	0.0411	0.0720
	500	-0.0410	0.0628	-0.0010	0.0677	-0.0502	0.0680	0.0333	0.0583	0.0328	0.0560
	1000	-0.0287	0.0457	0.0026	0.0502	-0.0353	0.0490	0.0231	0.0413	0.0231	0.0411
1.7	150	-0.1234	0.1439	-0.0425	0.1449	-0.1358	0.1607	0.0625	0.1071	0.0696	0.1083
	300	-0.0824	0.1015	-0.0183	0.1028	-0.0844	0.1050	0.0385	0.0715	0.0471	0.0762
	500	-0.0629	0.0781	-0.0084	0.0786	-0.0617	0.0777	0.0302	0.0558	0.0369	0.0601
	1000	-0.0432	0.0545	-0.0028	0.0537	-0.0395	0.0527	0.0213	0.0404	0.0275	0.0441
1.9	150	-0.1469	0.1684	-0.0405	0.1462	-0.2170	0.2406	0.0621	0.1043	0.0689	0.1055
	300	-0.0953	0.1122	-0.0131	0.0960	-0.1537	0.1724	0.0354	0.0687	0.0481	0.0750
	500	-0.0723	0.0853	-0.0094	0.0722	-0.1211	0.1359	0.0294	0.0546	0.0377	0.0594
	1000	-0.0508	0.0610	-0.0036	0.0514	-0.0865	0.0976	0.0202	0.0395	0.0293	0.0457

[a] The data generation is $(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 25: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$ and $m = \lfloor n^{.5} \rfloor$

			fextLWF		fextLPWF		ModLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	0.0531	0.2165	-0.1395	0.4262	0.0531	0.2257	0.0588	0.2184	-0.0381	0.2571
		300	0.0307	0.1746	-0.0790	0.3024	0.0323	0.1776	0.0348	0.1701	-0.0384	0.1945
		500	0.0223	0.1464	-0.0561	0.2423	0.0278	0.1479	0.0221	0.1438	-0.0278	0.1626
		1000	0.0137	0.1155	-0.0398	0.1752	0.0244	0.1197	0.0157	0.1181	-0.0259	0.1318
	.7	150	0.0347	0.2131	-0.1275	0.4308	0.0287	0.2208	0.0525	0.2072	-0.0282	0.2565
		300	0.0151	0.1690	-0.0888	0.3148	0.0101	0.1713	0.0206	0.1513	-0.0331	0.1916
		500	0.0047	0.1412	-0.0599	0.2557	0.0083	0.1422	0.0141	0.1267	-0.0295	0.1576
		1000	0.0049	0.1127	-0.0453	0.1935	0.0026	0.1140	0.0020	0.1014	-0.0230	0.1206
	.9	150	0.0184	0.2130	-0.1355	0.4467	0.0217	0.2195	0.0285	0.2085	-0.0096	0.2565
		300	0.0030	0.1666	-0.0925	0.3319	0.0040	0.1673	0.0043	0.1626	-0.0165	0.1945
		500	-0.0041	0.1420	-0.0648	0.2652	-0.0041	0.1416	-0.0043	0.1405	-0.0170	0.1578
		1000	-0.0079	0.1139	-0.0415	0.2012	-0.0021	0.1124	-0.0052	0.1120	-0.0150	0.1241
	1.1	150	0.0211	0.2244	-0.1206	0.4530	0.0176	0.2254	0.0203	0.2223	0.0137	0.2428
		300	0.0042	0.1700	-0.0721	0.3365	0.0018	0.1754	0.0079	0.1704	-0.0037	0.1748
		500	-0.0076	0.1419	-0.0571	0.2764	-0.0040	0.1431	-0.0015	0.1406	-0.0041	0.1464
		1000	-0.0056	0.1136	-0.0351	0.2013	-0.0060	0.1134	-0.0031	0.1125	-0.0052	0.1134
	1.3	150	0.0328	0.2318	-0.0699	0.4401	0.0223	0.2198	0.0222	0.2365	0.0255	0.2302
		300	0.0125	0.1801	-0.0553	0.3297	0.0085	0.1691	0.0037	0.1828	0.0109	0.1699
		500	0.0085	0.1508	-0.0350	0.2641	0.0015	0.1424	-0.0043	0.1511	0.0050	0.1410
		1000	0.0008	0.1179	-0.0366	0.1978	-0.0013	0.1138	-0.0002	0.1145	-0.0029	0.1113
	1.5	150	0.0309	0.2193	-0.0752	0.4243	0.0262	0.2191	0.0098	0.2449	0.0391	0.2204
		300	0.0185	0.1736	-0.0371	0.3080	0.0129	0.1720	-0.0028	0.1875	0.0177	0.1668
		500	0.0153	0.1450	-0.0294	0.2495	0.0091	0.1424	-0.0103	0.1588	0.0102	0.1388
		1000	0.0155	0.1192	-0.0445	0.1824	0.0073	0.1151	-0.0069	0.1249	0.0036	0.1126
	1.7	150	0.0221	0.2135	-0.0686	0.4117	0.0135	0.2061	0.0227	0.2283	0.0487	0.2186
		300	0.0098	0.1650	-0.0490	0.2971	0.0165	0.1625	0.0050	0.1736	0.0220	0.1716
		500	0.0052	0.1409	-0.0387	0.2387	0.0204	0.1401	0.0015	0.1418	0.0206	0.1388
		1000	0.0014	0.1116	-0.0757	0.1998	0.0183	0.1164	-0.0019	0.1118	0.0102	0.1109
	1.9	150	0.0126	0.2140	-0.0733	0.4236	-0.0355	0.1602	0.0223	0.2259	0.0448	0.2107
		300	-0.0027	0.1697	-0.0590	0.3026	-0.0178	0.1274	0.0017	0.1730	0.0230	0.1586
		500	0.0005	0.1406	-0.0493	0.2572	-0.0088	0.1114	0.0001	0.1409	0.0172	0.1344
		1000	-0.0070	0.1141	-0.0481	0.2126	0.0001	0.0921	-0.0067	0.1132	0.0082	0.1103

[a] The data generation is $(1 + .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 26: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$ and $m = \lfloor n^{.65} \rfloor$

			fextLWF		fextLPWF		ModLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	0.1226	0.1814	-0.0237	0.2261	0.1264	0.1832	0.1483	0.1933	0.1124	0.1813
		300	0.0866	0.1325	-0.0085	0.1579	0.0933	0.1357	0.1021	0.1386	0.0768	0.1281
		500	0.0627	0.1034	-0.0077	0.1271	0.0700	0.1066	0.0778	0.1110	0.0567	0.1038
		1000	0.0416	0.0749	-0.0005	0.0918	0.0500	0.0797	0.0530	0.0837	0.0373	0.0775
	.7	150	0.1125	0.1734	-0.0208	0.2256	0.1116	0.1721	0.1346	0.1882	0.1169	0.1897
		300	0.0776	0.1240	-0.0178	0.1624	0.0765	0.1221	0.0868	0.1296	0.0746	0.1297
		500	0.0561	0.0965	-0.0201	0.1421	0.0562	0.0968	0.0596	0.0988	0.0507	0.0997
		1000	0.0372	0.0708	-0.0086	0.1005	0.0380	0.0706	0.0391	0.0714	0.0330	0.0717
	.9	150	0.1029	0.1678	-0.0285	0.2515	0.1038	0.1680	0.1335	0.1896	0.1292	0.1903
		300	0.0696	0.1189	-0.0186	0.1681	0.0704	0.1194	0.0869	0.1310	0.0831	0.1309
		500	0.0527	0.0926	-0.0170	0.1371	0.0513	0.0927	0.0615	0.1002	0.0616	0.1009
		1000	0.0334	0.0691	-0.0097	0.0969	0.0317	0.0676	0.0376	0.0702	0.0362	0.0704
	1.1	150	0.1052	0.1685	-0.0168	0.2628	0.0981	0.1635	0.1333	0.1887	0.1323	0.1878
		300	0.0672	0.1167	-0.0133	0.1700	0.0656	0.1163	0.0882	0.1314	0.0844	0.1282
		500	0.0481	0.0914	-0.0167	0.1361	0.0514	0.0935	0.0591	0.0975	0.0607	0.0995
		1000	0.0315	0.0672	-0.0100	0.0992	0.0302	0.0676	0.0376	0.0707	0.0388	0.0715
	1.3	150	0.1120	0.1770	-0.0029	0.2642	0.0990	0.1622	0.1376	0.1924	0.1404	0.1908
		300	0.0779	0.1297	0.0051	0.1758	0.0670	0.1167	0.0893	0.1329	0.0855	0.1281
		500	0.0539	0.1010	-0.0005	0.1455	0.0498	0.0928	0.0648	0.1020	0.0649	0.1014
		1000	0.0333	0.0704	0.0004	0.1039	0.0321	0.0680	0.0405	0.0706	0.0425	0.0728
	1.5	150	0.1015	0.1641	-0.0044	0.2556	0.0854	0.1576	0.1340	0.1940	0.1443	0.1927
		300	0.0806	0.1250	0.0096	0.1573	0.0689	0.1181	0.0885	0.1348	0.0939	0.1341
		500	0.0616	0.0991	0.0065	0.1247	0.0539	0.0965	0.0654	0.1039	0.0665	0.1020
		1000	0.0443	0.0748	0.0058	0.0948	0.0376	0.0722	0.0425	0.0739	0.0447	0.0740
	1.7	150	0.0825	0.1534	-0.0167	0.2445	0.0662	0.1447	0.1347	0.1900	0.1445	0.1920
		300	0.0638	0.1148	-0.0079	0.1566	0.0643	0.1151	0.0880	0.1307	0.0964	0.1350
		500	0.0491	0.0921	-0.0084	0.1310	0.0549	0.0951	0.0605	0.0993	0.0718	0.1060
		1000	0.0335	0.0678	-0.0045	0.0977	0.0453	0.0763	0.0396	0.0724	0.0471	0.0760
	1.9	150	0.0757	0.1500	-0.0252	0.2613	-0.0189	0.1125	0.1355	0.1902	0.1261	0.1744
		300	0.0577	0.1113	-0.0148	0.1624	0.0082	0.0807	0.0844	0.1292	0.0888	0.1268
		500	0.0445	0.0894	-0.0165	0.1352	0.0171	0.0686	0.0597	0.0978	0.0653	0.0976
		1000	0.0288	0.0654	-0.0101	0.0971	0.0248	0.0583	0.0390	0.0711	0.0439	0.0730

[a] The data generation is $(1 + .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 27: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$ and $m = \lfloor n^{.8} \rfloor$

			fextLWF		fextLPWF		modLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	0.2468	0.2596	0.1209	0.1793	0.2464	0.2593	0.3741	0.3857	0.3725	0.3842
		300	0.2173	0.2255	0.0898	0.1284	0.2162	0.2241	0.2965	0.3041	0.2921	0.3002
		500	0.1959	0.2015	0.0700	0.1020	0.1945	0.2002	0.2547	0.2598	0.2527	0.2583
		1000	0.1661	0.1698	0.0499	0.0736	0.1660	0.1698	0.2052	0.2085	0.2041	0.2076
	.7	150	0.2286	0.2424	0.1127	0.1741	0.2230	0.2372	0.3746	0.3860	0.3722	0.3836
		300	0.2010	0.2096	0.0805	0.1239	0.2007	0.2091	0.2945	0.3015	0.2944	0.3014
		500	0.1831	0.1887	0.0635	0.0965	0.1825	0.1882	0.2544	0.2593	0.2545	0.2592
		1000	0.1565	0.1602	0.0454	0.0697	0.1568	0.1606	0.2058	0.2089	0.2059	0.2092
	.9	150	0.2130	0.2279	0.1044	0.1707	0.2061	0.2216	0.3697	0.3806	0.3699	0.3810
		300	0.1896	0.1982	0.0745	0.1191	0.1873	0.1962	0.2936	0.3007	0.2928	0.3000
		500	0.1743	0.1804	0.0617	0.0933	0.1736	0.1796	0.2536	0.2585	0.2537	0.2587
		1000	0.1505	0.1543	0.0430	0.0683	0.1505	0.1545	0.2052	0.2084	0.2066	0.2097
	1.1	150	0.1978	0.2134	0.1115	0.1748	0.1920	0.2082	0.3721	0.3832	0.3719	0.3834
		300	0.1811	0.1905	0.0746	0.1182	0.1788	0.1883	0.2929	0.2999	0.2937	0.3009
		500	0.1657	0.1722	0.0586	0.0924	0.1639	0.1702	0.2521	0.2572	0.2530	0.2577
		1000	0.1454	0.1492	0.0419	0.0674	0.1445	0.1486	0.2061	0.2094	0.2059	0.2089
	1.3	150	0.1811	0.1973	0.1113	0.1681	0.1743	0.1930	0.3726	0.3836	0.3740	0.3846
		300	0.1753	0.1851	0.0843	0.1263	0.1688	0.1787	0.2944	0.3014	0.2942	0.3012
		500	0.1639	0.1717	0.0660	0.1021	0.1583	0.1649	0.2555	0.2605	0.2540	0.2589
		1000	0.1444	0.1494	0.0453	0.0718	0.1392	0.1435	0.2070	0.2101	0.2068	0.2100
	1.5	150	0.1495	0.1711	0.0894	0.1616	0.1279	0.1584	0.3748	0.3861	0.3730	0.3837
		300	0.1545	0.1661	0.0743	0.1211	0.1422	0.1557	0.2957	0.3025	0.2946	0.3014
		500	0.1499	0.1573	0.0603	0.0949	0.1433	0.1512	0.2560	0.2610	0.2567	0.2615
		1000	0.1365	0.1407	0.0467	0.0719	0.1325	0.1369	0.2072	0.2104	0.2071	0.2101
	1.7	150	0.1191	0.1490	0.0785	0.1729	0.0450	0.1277	0.3709	0.3823	0.3630	0.3734
		300	0.1385	0.1511	0.0710	0.1265	0.0879	0.1216	0.2935	0.3007	0.2932	0.2998
		500	0.1376	0.1454	0.0609	0.0981	0.1027	0.1202	0.2529	0.2578	0.2548	0.2597
		1000	0.1270	0.1316	0.0457	0.0694	0.1097	0.1170	0.2063	0.2094	0.2086	0.2117
	1.9	150	0.0982	0.1348	0.0918	0.1721	-0.1106	0.1996	0.3704	0.3814	0.3188	0.3321
		300	0.1237	0.1381	0.0732	0.1192	-0.0537	0.1435	0.2917	0.2986	0.2603	0.2687
		500	0.1256	0.1343	0.0609	0.0941	-0.0226	0.1150	0.2530	0.2579	0.2284	0.2339
		1000	0.1190	0.1240	0.0431	0.0674	0.0068	0.0879	0.2053	0.2084	0.1910	0.1943

[a] The data generation is $(1 + .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 28: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .4$ and $m = \lfloor n^{.5} \rfloor$

			fextLWF		fextLPWF		modLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	0.0282	0.2270	-0.1588	0.4181	0.0243	0.2285	-0.0067	0.2164	-0.1046	0.2659
		300	0.0296	0.1860	-0.0946	0.2955	0.0360	0.1830	-0.0020	0.1701	-0.0676	0.2019
		500	0.0248	0.1587	-0.0680	0.2372	0.0411	0.1578	0.0065	0.1428	-0.0526	0.1695
		1000	0.0196	0.1304	-0.0475	0.1722	0.0353	0.1302	0.0092	0.1166	-0.0343	0.1295
	.7	150	-0.0114	0.2077	-0.1484	0.4251	-0.0186	0.2222	-0.0066	0.1975	-0.0954	0.2694
		300	-0.0041	0.1677	-0.1045	0.3142	-0.0105	0.1724	-0.0067	0.1508	-0.0656	0.2003
		500	-0.0025	0.1414	-0.0711	0.2570	-0.0003	0.1387	-0.0067	0.1262	-0.0484	0.1611
		1000	-0.0045	0.1141	-0.0519	0.1969	0.0011	0.1125	-0.0054	0.1032	-0.0360	0.1249
	.9	150	-0.0326	0.2124	-0.1543	0.4442	-0.0368	0.2212	-0.0319	0.2039	-0.0812	0.2689
		300	-0.0237	0.1682	-0.1005	0.3320	-0.0260	0.1714	-0.0246	0.1632	-0.0508	0.1958
		500	-0.0175	0.1407	-0.0731	0.2685	-0.0185	0.1433	-0.0191	0.1378	-0.0394	0.1601
		1000	-0.0106	0.1138	-0.0414	0.2016	-0.0138	0.1136	-0.0147	0.1135	-0.0219	0.1252
	1.1	150	-0.0344	0.2178	-0.1442	0.4444	-0.0386	0.2229	-0.0356	0.2127	-0.0503	0.2514
		300	-0.0224	0.1699	-0.0818	0.3316	-0.0243	0.1672	-0.0285	0.1719	-0.0310	0.1824
		500	-0.0183	0.1424	-0.0628	0.2737	-0.0201	0.1438	-0.0197	0.1450	-0.0239	0.1477
		1000	-0.0117	0.1132	-0.0362	0.2012	-0.0127	0.1121	-0.0120	0.1125	-0.0142	0.1141
	1.3	150	-0.0291	0.2324	-0.1015	0.4255	-0.0434	0.2251	-0.0425	0.2367	-0.0265	0.2265
		300	-0.0110	0.1765	-0.0724	0.3216	-0.0183	0.1701	-0.0254	0.1780	-0.0210	0.1739
		500	-0.0138	0.1486	-0.0464	0.2547	-0.0118	0.1412	-0.0202	0.1508	-0.0105	0.1411
		1000	-0.0085	0.1201	-0.0391	0.1946	-0.0111	0.1132	-0.0098	0.1163	-0.0118	0.1131
	1.5	150	-0.0140	0.2230	-0.1023	0.4172	-0.0310	0.2197	-0.0715	0.2698	-0.0220	0.2203
		300	-0.0022	0.1800	-0.0555	0.3058	-0.0163	0.1697	-0.0493	0.2189	-0.0065	0.1676
		500	0.0051	0.1521	-0.0402	0.2481	-0.0099	0.1465	-0.0334	0.1838	-0.0068	0.1396
		1000	0.0129	0.1270	-0.0509	0.1808	-0.0007	0.1155	-0.0168	0.1406	-0.0024	0.1098
	1.7	150	-0.0260	0.2120	-0.0874	0.4078	-0.0227	0.2063	-0.0509	0.2457	-0.0153	0.2137
		300	-0.0101	0.1694	-0.0572	0.2980	-0.0028	0.1635	-0.0296	0.1843	-0.0027	0.1639
		500	-0.0052	0.1388	-0.0441	0.2403	0.0042	0.1399	-0.0178	0.1482	-0.0004	0.1415
		1000	-0.0020	0.1119	-0.0824	0.2026	0.0087	0.1150	-0.0143	0.1172	0.0049	0.1114
	1.9	150	-0.0356	0.2177	-0.0900	0.4178	-0.0661	0.1812	-0.0423	0.2334	-0.0022	0.2113
		300	-0.0271	0.1665	-0.0635	0.2977	-0.0364	0.1388	-0.0279	0.1749	0.0016	0.1613
		500	-0.0172	0.1403	-0.0491	0.2549	-0.0196	0.1161	-0.0185	0.1448	0.0023	0.1376
		1000	-0.0133	0.1130	-0.0511	0.2149	-0.0064	0.0937	-0.0136	0.1166	0.0038	0.1104

[a] The data generation is $(1 - .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 29: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .4$ and $m = \lfloor n^{.65} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	-0.0342	0.1434	-0.0570	0.2190	-0.0062	0.1415	-0.0145	0.1396	-0.0906	0.1740
		300	-0.0184	0.1020	-0.0331	0.1490	0.0065	0.1051	-0.0016	0.1021	-0.0495	0.1203
		500	-0.0068	0.0846	-0.0245	0.1143	0.0115	0.0891	-0.0016	0.0852	-0.0341	0.0932
		1000	-0.0027	0.0634	-0.0155	0.0827	0.0156	0.0696	0.0016	0.0665	-0.0187	0.0687
	.7	150	-0.0469	0.1383	-0.0613	0.2311	-0.0444	0.1382	-0.0278	0.1220	-0.0910	0.1717
		300	-0.0276	0.1044	-0.0427	0.1710	-0.0254	0.1000	-0.0164	0.0906	-0.0606	0.1176
		500	-0.0186	0.0828	-0.0436	0.1460	-0.0172	0.0793	-0.0131	0.0730	-0.0449	0.0937
		1000	-0.0078	0.0621	-0.0142	0.1078	-0.0064	0.0600	-0.0086	0.0562	-0.0309	0.0715
	.9	150	-0.0600	0.1421	-0.0627	0.2539	-0.0622	0.1439	-0.0401	0.1388	-0.0837	0.1776
		300	-0.0350	0.1026	-0.0351	0.1738	-0.0367	0.1013	-0.0275	0.1030	-0.0485	0.1248
		500	-0.0248	0.0817	-0.0224	0.1336	-0.0265	0.0824	-0.0180	0.0828	-0.0284	0.0916
		1000	-0.0153	0.0609	-0.0137	0.0971	-0.0157	0.0610	-0.0111	0.0625	-0.0161	0.0663
	1.1	150	-0.0658	0.1472	-0.0476	0.2519	-0.0670	0.1436	-0.0432	0.1407	-0.0591	0.1520
		300	-0.0407	0.1036	-0.0290	0.1760	-0.0368	0.1023	-0.0237	0.0990	-0.0363	0.1035
		500	-0.0278	0.0831	-0.0214	0.1349	-0.0272	0.0821	-0.0170	0.0806	-0.0254	0.0822
		1000	-0.0157	0.0613	-0.0125	0.0958	-0.0167	0.0609	-0.0105	0.0597	-0.0148	0.0612
	1.3	150	-0.0611	0.1476	-0.0345	0.2425	-0.0625	0.1464	-0.0374	0.1435	-0.0455	0.1377
		300	-0.0353	0.1042	-0.0159	0.1726	-0.0357	0.1028	-0.0211	0.0996	-0.0276	0.0988
		500	-0.0237	0.0818	-0.0095	0.1352	-0.0237	0.0804	-0.0137	0.0795	-0.0213	0.0810
		1000	-0.0135	0.0607	-0.0061	0.0987	-0.0142	0.0616	-0.0088	0.0618	-0.0131	0.0604
	1.5	150	-0.0568	0.1465	-0.0359	0.2233	-0.0646	0.1481	-0.0516	0.1686	-0.0475	0.1373
		300	-0.0218	0.1054	-0.0169	0.1590	-0.0316	0.1017	-0.0255	0.1146	-0.0233	0.0977
		500	-0.0072	0.0859	-0.0191	0.1206	-0.0205	0.0829	-0.0129	0.0885	-0.0157	0.0780
		1000	0.0043	0.0673	-0.0145	0.0902	-0.0081	0.0629	-0.0037	0.0639	-0.0083	0.0593
	1.7	150	-0.0733	0.1441	-0.0441	0.2237	-0.0690	0.1473	-0.0368	0.1418	-0.0401	0.1342
		300	-0.0398	0.1034	-0.0289	0.1629	-0.0290	0.1008	-0.0200	0.1017	-0.0196	0.0989
		500	-0.0259	0.0829	-0.0469	0.1473	-0.0130	0.0811	-0.0133	0.0816	-0.0090	0.0790
		1000	-0.0127	0.0605	-0.0522	0.1313	-0.0012	0.0638	-0.0077	0.0597	-0.0016	0.0598
	1.9	150	-0.0893	0.1556	-0.0490	0.2350	-0.1011	0.1544	-0.0413	0.1398	-0.0303	0.1305
		300	-0.0513	0.1091	-0.0498	0.1883	-0.0538	0.1030	-0.0218	0.0999	-0.0131	0.0964
		500	-0.0346	0.0849	-0.0502	0.1723	-0.0305	0.0794	-0.0167	0.0802	-0.0055	0.0809
		1000	-0.0201	0.0633	-0.0140	0.1014	-0.0112	0.0581	-0.0109	0.0594	0.0000	0.0614

[a] The data generation is $(1 - .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 30: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .4$ and $m = \lfloor n^{.8} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	-0.1800	0.1988	-0.0106	0.1228	-0.1562	0.1798	-0.0189	0.1365	-0.1312	0.1601
		300	-0.1218	0.1363	-0.0073	0.0860	-0.1055	0.1246	-0.0096	0.1025	-0.0920	0.1129
		500	-0.0956	0.1069	-0.0021	0.0675	-0.0823	0.0983	-0.0004	0.0853	-0.0694	0.0853
		1000	-0.0692	0.0779	-0.0002	0.0485	-0.0581	0.0705	0.0025	0.0682	-0.0501	0.0619
	.7	150	-0.1892	0.2035	-0.0188	0.1398	-0.1909	0.2087	-0.0278	0.1227	-0.1152	0.1469
		300	-0.1369	0.1495	-0.0116	0.1036	-0.1332	0.1461	-0.0187	0.0886	-0.0802	0.1001
		500	-0.1066	0.1174	-0.0043	0.0797	-0.1047	0.1147	-0.0123	0.0741	-0.0637	0.0782
		1000	-0.0763	0.0838	0.0006	0.0575	-0.0766	0.0838	-0.0084	0.0566	-0.0454	0.0563
	.9	150	-0.2125	0.2270	-0.0161	0.1339	-0.2135	0.2280	-0.0399	0.1354	-0.1220	0.1511
		300	-0.1469	0.1582	-0.0102	0.0955	-0.1478	0.1591	-0.0252	0.1029	-0.0827	0.1044
		500	-0.1171	0.1255	-0.0079	0.0741	-0.1183	0.1269	-0.0169	0.0835	-0.0637	0.0799
		1000	-0.0853	0.0915	-0.0020	0.0523	-0.0840	0.0906	-0.0085	0.0610	-0.0452	0.0570
	1.1	150	-0.2257	0.2404	-0.0174	0.1352	-0.2293	0.2435	-0.0426	0.1410	-0.1143	0.1432
		300	-0.1561	0.1662	-0.0096	0.0914	-0.1584	0.1688	-0.0221	0.0987	-0.0794	0.0995
		500	-0.1254	0.1334	-0.0073	0.0732	-0.1259	0.1343	-0.0166	0.0800	-0.0626	0.0789
		1000	-0.0895	0.0958	-0.0034	0.0510	-0.0909	0.0970	-0.0102	0.0594	-0.0450	0.0567
	1.3	150	-0.2346	0.2486	-0.0099	0.1322	-0.2356	0.2493	-0.0393	0.1446	-0.1099	0.1382
		300	-0.1619	0.1725	-0.0019	0.0971	-0.1632	0.1736	-0.0210	0.1013	-0.0758	0.0970
		500	-0.1281	0.1366	-0.0014	0.0742	-0.1287	0.1368	-0.0130	0.0821	-0.0601	0.0761
		1000	-0.0939	0.0999	0.0002	0.0532	-0.0935	0.0995	-0.0082	0.0599	-0.0439	0.0555
	1.5	150	-0.2404	0.2552	-0.0164	0.1237	-0.2461	0.2605	-0.0505	0.1663	-0.1038	0.1334
		300	-0.1644	0.1767	-0.0063	0.0885	-0.1689	0.1797	-0.0227	0.1152	-0.0741	0.0955
		500	-0.1292	0.1396	-0.0032	0.0699	-0.1334	0.1418	-0.0133	0.0908	-0.0577	0.0737
		1000	-0.0900	0.0993	0.0014	0.0495	-0.0949	0.1012	-0.0016	0.0639	-0.0414	0.0536
	1.7	150	-0.2443	0.2563	-0.0402	0.1466	-0.2595	0.2735	-0.0381	0.1455	-0.0970	0.1285
		300	-0.1769	0.1845	-0.0164	0.1046	-0.1758	0.1864	-0.0192	0.1001	-0.0676	0.0903
		500	-0.1443	0.1513	-0.0038	0.0775	-0.1352	0.1438	-0.0138	0.0796	-0.0521	0.0705
		1000	-0.1042	0.1099	0.0001	0.0537	-0.0964	0.1031	-0.0076	0.0610	-0.0369	0.0510
	1.9	150	-0.2872	0.2980	-0.0350	0.1478	-0.3047	0.3167	-0.0368	0.1356	-0.0871	0.1207
		300	-0.2006	0.2095	-0.0161	0.0962	-0.2164	0.2254	-0.0263	0.1009	-0.0583	0.0842
		500	-0.1562	0.1631	-0.0091	0.0725	-0.1743	0.1815	-0.0163	0.0802	-0.0461	0.0677
		1000	-0.1127	0.1179	-0.0031	0.0509	-0.1270	0.1323	-0.0092	0.0593	-0.0308	0.0486

[a] The data generation is $(1 - .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 31: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1,d,0)$ Model with $\phi = .8$ and $m = \lfloor n^{.5} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	0.0773	0.2686	-0.1774	0.4050	0.0874	0.2528	-0.0089	0.2352	-0.1036	0.2805
		300	0.0641	0.2191	-0.1146	0.2915	0.0862	0.2144	-0.0025	0.1810	-0.0714	0.2195
		500	0.0562	0.1874	-0.0857	0.2336	0.0872	0.1866	-0.0026	0.1539	-0.0517	0.1760
		1000	0.0556	0.1601	-0.0599	0.1720	0.0804	0.1586	0.0037	0.1200	-0.0376	0.1345
	.7	150	0.0019	0.2134	-0.1825	0.4328	-0.0026	0.2208	-0.0261	0.2043	-0.1111	0.2761
		300	0.0039	0.1697	-0.1208	0.3050	0.0118	0.1705	-0.0162	0.1625	-0.0734	0.2039
		500	0.0057	0.1461	-0.0967	0.2594	0.0037	0.1483	-0.0168	0.1332	-0.0560	0.1658
		1000	0.0050	0.1185	-0.0583	0.1993	0.0080	0.1163	-0.0137	0.1064	-0.0434	0.1309
	.9	150	-0.0302	0.2160	-0.1641	0.4400	-0.0396	0.2236	-0.0507	0.2061	-0.0990	0.2735
		300	-0.0219	0.1679	-0.1005	0.3313	-0.0224	0.1702	-0.0348	0.1666	-0.0604	0.2021
		500	-0.0187	0.1428	-0.0799	0.2743	-0.0176	0.1455	-0.0247	0.1405	-0.0417	0.1620
		1000	-0.0094	0.1127	-0.0347	0.1986	-0.0102	0.1145	-0.0182	0.1153	-0.0319	0.1285
	1.1	150	-0.0406	0.2237	-0.1532	0.4411	-0.0475	0.2254	-0.0569	0.2180	-0.0712	0.2545
		300	-0.0274	0.1765	-0.0959	0.3323	-0.0333	0.1751	-0.0329	0.1689	-0.0408	0.1868
		500	-0.0207	0.1451	-0.0700	0.2748	-0.0229	0.1423	-0.0240	0.1438	-0.0305	0.1507
		1000	-0.0137	0.1137	-0.0408	0.1991	-0.0077	0.1111	-0.0190	0.1137	-0.0158	0.1145
	1.3	150	-0.0362	0.2255	-0.1372	0.4182	-0.0431	0.2244	-0.0553	0.2384	-0.0422	0.2270
		300	-0.0177	0.1732	-0.0858	0.3175	-0.0260	0.1717	-0.0388	0.1878	-0.0272	0.1702
		500	-0.0128	0.1453	-0.0628	0.2566	-0.0137	0.1422	-0.0261	0.1505	-0.0191	0.1388
		1000	-0.0082	0.1162	-0.0324	0.1882	-0.0090	0.1132	-0.0177	0.1162	-0.0132	0.1110
	1.5	150	-0.0328	0.2280	-0.1415	0.4059	-0.0335	0.2199	-0.1094	0.3122	-0.0377	0.2212
		300	-0.0122	0.1814	-0.0843	0.2973	-0.0150	0.1719	-0.0743	0.2608	-0.0167	0.1671
		500	0.0014	0.1559	-0.0644	0.2387	-0.0049	0.1446	-0.0513	0.2310	-0.0131	0.1361
		1000	0.0137	0.1326	-0.0650	0.1747	0.0015	0.1142	-0.0215	0.1826	-0.0074	0.1100
	1.7	150	-0.0318	0.2266	-0.1235	0.4179	-0.0328	0.2109	-0.0736	0.2715	-0.0311	0.2155
		300	-0.0122	0.1727	-0.0744	0.3104	-0.0064	0.1665	-0.0388	0.2012	-0.0094	0.1680
		500	0.0009	0.1418	-0.0518	0.2500	0.0060	0.1428	-0.0233	0.1645	-0.0018	0.1414
		1000	0.0030	0.1158	-0.0939	0.2068	0.0110	0.1149	-0.0131	0.1213	0.0020	0.1130
	1.9	150	-0.0422	0.2188	-0.1067	0.4214	-0.0674	0.1800	-0.0618	0.2374	-0.0143	0.2149
		300	-0.0269	0.1698	-0.0626	0.3071	-0.0320	0.1388	-0.0401	0.1803	-0.0020	0.1671
		500	-0.0191	0.1413	-0.0469	0.2532	-0.0154	0.1150	-0.0301	0.1448	0.0016	0.1427
		1000	-0.0143	0.1127	-0.0588	0.2196	-0.0040	0.0953	-0.0170	0.1120	0.0062	0.1157

[a] The data generation is $(1-.8L)(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 32: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .8$ and $m = \lfloor n^{.65} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	-0.0173	0.1645	-0.0725	0.2078	0.0274	0.1561	-0.0395	0.1404	-0.0883	0.1729
		300	0.0004	0.1229	-0.0407	0.1423	0.0414	0.1275	-0.0194	0.1046	-0.0538	0.1220
		500	0.0018	0.1014	-0.0316	0.1194	0.0408	0.1103	-0.0098	0.0846	-0.0363	0.0952
		1000	0.0075	0.0790	-0.0163	0.0845	0.0396	0.0916	-0.0025	0.0666	-0.0213	0.0701
	.7	150	-0.0428	0.1460	-0.0941	0.2370	-0.0375	0.1397	-0.0486	0.1310	-0.0897	0.1677
		300	-0.0248	0.1073	-0.0638	0.1806	-0.0187	0.1015	-0.0330	0.0959	-0.0588	0.1180
		500	-0.0146	0.0882	-0.0496	0.1578	-0.0102	0.0820	-0.0256	0.0804	-0.0452	0.0947
		1000	-0.0051	0.0651	-0.0275	0.1215	-0.0049	0.0619	-0.0174	0.0631	-0.0296	0.0714
	.9	150	-0.0653	0.1469	-0.0764	0.2667	-0.0676	0.1471	-0.0610	0.1481	-0.0777	0.1726
		300	-0.0388	0.1038	-0.0459	0.1835	-0.0384	0.1023	-0.0393	0.1096	-0.0443	0.1207
		500	-0.0259	0.0815	-0.0293	0.1523	-0.0292	0.0824	-0.0272	0.0878	-0.0315	0.0952
		1000	-0.0171	0.0619	-0.0130	0.1024	-0.0172	0.0609	-0.0143	0.0634	-0.0156	0.0648
	1.1	150	-0.0757	0.1518	-0.0618	0.2691	-0.0745	0.1507	-0.0604	0.1485	-0.0626	0.1549
		300	-0.0469	0.1066	-0.0312	0.1755	-0.0452	0.1067	-0.0351	0.1030	-0.0356	0.1013
		500	-0.0281	0.0830	-0.0201	0.1379	-0.0322	0.0850	-0.0225	0.0810	-0.0250	0.0836
		1000	-0.0197	0.0634	-0.0123	0.0999	-0.0181	0.0619	-0.0144	0.0608	-0.0150	0.0610
	1.3	150	-0.0711	0.1492	-0.0476	0.2566	-0.0765	0.1513	-0.0562	0.1494	-0.0512	0.1397
		300	-0.0428	0.1062	-0.0155	0.1672	-0.0419	0.1051	-0.0327	0.1047	-0.0304	0.0994
		500	-0.0283	0.0837	-0.0093	0.1358	-0.0273	0.0825	-0.0214	0.0816	-0.0192	0.0795
		1000	-0.0171	0.0624	-0.0062	0.1014	-0.0140	0.0619	-0.0126	0.0615	-0.0115	0.0600
	1.5	150	-0.0674	0.1575	-0.0725	0.2653	-0.0710	0.1479	-0.0849	0.2087	-0.0440	0.1375
		300	-0.0242	0.1173	-0.0300	0.1545	-0.0402	0.1069	-0.0320	0.1461	-0.0262	0.0987
		500	-0.0044	0.0998	-0.0213	0.1204	-0.0233	0.0833	-0.0114	0.1097	-0.0185	0.0798
		1000	0.0098	0.0817	-0.0096	0.0883	-0.0099	0.0617	0.0015	0.0764	-0.0087	0.0591
	1.7	150	-0.0716	0.1515	-0.0658	0.2738	-0.0727	0.1494	-0.0561	0.1613	-0.0398	0.1367
		300	-0.0339	0.1048	-0.0314	0.1757	-0.0321	0.1022	-0.0264	0.1064	-0.0196	0.0969
		500	-0.0210	0.0834	-0.0246	0.1462	-0.0154	0.0807	-0.0176	0.0813	-0.0080	0.0779
		1000	-0.0085	0.0633	-0.0096	0.1086	-0.0024	0.0627	-0.0098	0.0609	-0.0022	0.0611
	1.9	150	-0.0957	0.1616	-0.0643	0.2848	-0.1079	0.1607	-0.0547	0.1460	-0.0295	0.1289
		300	-0.0531	0.1093	-0.0283	0.1705	-0.0567	0.1048	-0.0332	0.1026	-0.0159	0.0989
		500	-0.0352	0.0861	-0.0204	0.1341	-0.0336	0.0800	-0.0242	0.0819	-0.0076	0.0809
		1000	-0.0194	0.0622	-0.0109	0.0961	-0.0114	0.0576	-0.0136	0.0605	0.0016	0.0614

[a] The data generation is $(1 - .8L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 33: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .8$ and $m = \lfloor n^{.8} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	-0.2437	0.2581	-0.0157	0.1163	-0.1822	0.2126	-0.1794	0.1992	-0.2155	0.2353
		300	-0.1591	0.1706	-0.0081	0.0819	-0.1078	0.1369	-0.1137	0.1298	-0.1330	0.1478
		500	-0.1230	0.1322	-0.0066	0.0623	-0.0833	0.1075	-0.0864	0.0991	-0.1001	0.1113
		1000	-0.0862	0.0935	-0.0026	0.0460	-0.0551	0.0755	-0.0607	0.0704	-0.0686	0.0776
	.7	150	-0.2464	0.2591	-0.0302	0.1548	-0.2422	0.2581	-0.1574	0.1839	-0.1987	0.2224
		300	-0.1693	0.1798	-0.0206	0.1183	-0.1589	0.1710	-0.1003	0.1164	-0.1216	0.1376
		500	-0.1334	0.1435	-0.0124	0.0982	-0.1231	0.1328	-0.0789	0.0909	-0.0921	0.1036
		1000	-0.0914	0.0997	-0.0020	0.0712	-0.0882	0.0955	-0.0588	0.0678	-0.0657	0.0746
	.9	150	-0.2773	0.2897	-0.0103	0.1568	-0.2757	0.2890	-0.1822	0.1980	-0.2044	0.2237
		300	-0.1810	0.1906	-0.0050	0.1037	-0.1819	0.1916	-0.1222	0.1382	-0.1327	0.1502
		500	-0.1422	0.1496	-0.0008	0.0754	-0.1412	0.1484	-0.0920	0.1055	-0.0939	0.1078
		1000	-0.1011	0.1067	-0.0002	0.0530	-0.1004	0.1059	-0.0641	0.0731	-0.0645	0.0733
	1.1	150	-0.2923	0.3042	-0.0012	0.1413	-0.2933	0.3052	-0.1860	0.2062	-0.1918	0.2127
		300	-0.1916	0.2005	-0.0029	0.0939	-0.1950	0.2038	-0.1188	0.1339	-0.1185	0.1333
		500	-0.1494	0.1564	-0.0001	0.0730	-0.1488	0.1562	-0.0910	0.1025	-0.0927	0.1042
		1000	-0.1063	0.1115	-0.0026	0.0532	-0.1067	0.1121	-0.0642	0.0726	-0.0644	0.0731
	1.3	150	-0.3008	0.3126	0.0111	0.1333	-0.3024	0.3145	-0.1845	0.2045	-0.1832	0.2021
		300	-0.1990	0.2080	0.0043	0.0970	-0.2007	0.2096	-0.1173	0.1322	-0.1170	0.1315
		500	-0.1537	0.1610	0.0020	0.0751	-0.1548	0.1615	-0.0893	0.1013	-0.0905	0.1021
		1000	-0.1100	0.1153	0.0008	0.0535	-0.1110	0.1163	-0.0616	0.0707	-0.0616	0.0708
	1.5	150	-0.3087	0.3209	-0.0136	0.1242	-0.3134	0.3255	-0.1880	0.2169	-0.1790	0.1988
		300	-0.2009	0.2106	-0.0063	0.0900	-0.2043	0.2134	-0.1103	0.1340	-0.1138	0.1293
		500	-0.1554	0.1641	-0.0047	0.0692	-0.1573	0.1646	-0.0778	0.0967	-0.0860	0.0974
		1000	-0.1080	0.1161	-0.0010	0.0538	-0.1123	0.1179	-0.0516	0.0650	-0.0584	0.0679
	1.7	150	-0.3032	0.3172	-0.0278	0.1561	-0.3200	0.3321	-0.1804	0.2023	-0.1700	0.1911
		300	-0.2012	0.2094	-0.0124	0.1160	-0.2081	0.2175	-0.1127	0.1294	-0.1080	0.1238
		500	-0.1614	0.1682	-0.0039	0.0888	-0.1604	0.1679	-0.0855	0.0983	-0.0807	0.0941
		1000	-0.1175	0.1234	0.0062	0.0594	-0.1110	0.1171	-0.0609	0.0703	-0.0556	0.0660
	1.9	150	-0.3408	0.3487	-0.0196	0.1510	-0.3495	0.3594	-0.1836	0.2031	-0.1582	0.1811
		300	-0.2329	0.2406	-0.0064	0.0952	-0.2412	0.2486	-0.1193	0.1338	-0.0962	0.1149
		500	-0.1799	0.1860	-0.0033	0.0726	-0.1915	0.1977	-0.0900	0.1018	-0.0720	0.0881
		1000	-0.1287	0.1331	-0.0011	0.0522	-0.1384	0.1430	-0.0622	0.0707	-0.0478	0.0611

[a] The data generation is $(1 - .8L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 34: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$ and $m = \lfloor n^{.5} \rfloor$

			fextLWF		fextLPWF(r=1)		ModLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	-0.0066	0.2324	-0.1552	0.3987	0.0134	0.2273	-0.0379	0.2216	-0.1360	0.2777
		300	0.0128	0.1900	-0.1059	0.2894	0.0333	0.1843	-0.0192	0.1753	-0.0920	0.2162
		500	0.0194	0.1651	-0.0775	0.2335	0.0394	0.1619	-0.0089	0.1456	-0.0681	0.1737
		1000	0.0228	0.1351	-0.0513	0.1758	0.0428	0.1328	-0.0002	0.1165	-0.0414	0.1354
	.7	150	-0.0430	0.2112	-0.1696	0.4261	-0.0495	0.2246	-0.0426	0.2027	-0.1335	0.2856
		300	-0.0253	0.1685	-0.1135	0.3162	-0.0204	0.1711	-0.0262	0.1567	-0.0823	0.2074
		500	-0.0101	0.1414	-0.0822	0.2532	-0.0121	0.1443	-0.0193	0.1308	-0.0639	0.1676
		1000	-0.0042	0.1143	-0.0530	0.1980	-0.0017	0.1142	-0.0128	0.1039	-0.0430	0.1275
	.9	150	-0.0697	0.2225	-0.1480	0.4339	-0.0722	0.2310	-0.0638	0.2068	-0.1274	0.2811
		300	-0.0439	0.1733	-0.1076	0.3316	-0.0383	0.1792	-0.0381	0.1635	-0.0756	0.2076
		500	-0.0275	0.1451	-0.0774	0.2701	-0.0300	0.1450	-0.0353	0.1409	-0.0505	0.1659
		1000	-0.0159	0.1117	-0.0348	0.1993	-0.0162	0.1131	-0.0202	0.1132	-0.0323	0.1287
	1.1	150	-0.0798	0.2359	-0.1505	0.4435	-0.0839	0.2355	-0.0786	0.2242	-0.0926	0.2641
		300	-0.0464	0.1713	-0.0911	0.3343	-0.0437	0.1728	-0.0479	0.1763	-0.0528	0.1887
		500	-0.0263	0.1461	-0.0623	0.2718	-0.0327	0.1442	-0.0334	0.1440	-0.0360	0.1522
		1000	-0.0184	0.1129	-0.0301	0.1986	-0.0179	0.1150	-0.0223	0.1139	-0.0195	0.1153
	1.3	150	-0.0700	0.2346	-0.1371	0.4337	-0.0693	0.2298	-0.0825	0.2394	-0.0693	0.2373
		300	-0.0377	0.1813	-0.0752	0.3242	-0.0376	0.1749	-0.0474	0.1869	-0.0408	0.1727
		500	-0.0231	0.1483	-0.0482	0.2554	-0.0241	0.1428	-0.0306	0.1519	-0.0270	0.1432
		1000	-0.0129	0.1191	-0.0359	0.1938	-0.0118	0.1143	-0.0152	0.1156	-0.0142	0.1115
	1.5	150	-0.0607	0.2307	-0.1136	0.4192	-0.0552	0.2270	-0.1154	0.2892	-0.0616	0.2227
		300	-0.0194	0.1777	-0.0691	0.3036	-0.0304	0.1751	-0.0738	0.2373	-0.0342	0.1679
		500	-0.0018	0.1559	-0.0468	0.2472	-0.0143	0.1430	-0.0537	0.1968	-0.0175	0.1422
		1000	0.0086	0.1270	-0.0486	0.1775	-0.0026	0.1139	-0.0225	0.1524	-0.0114	0.1104
	1.7	150	-0.0615	0.2186	-0.0957	0.4150	-0.0543	0.2307	-0.0876	0.2606	-0.0505	0.2216
		300	-0.0286	0.1702	-0.0654	0.3016	-0.0171	0.1733	-0.0461	0.1897	-0.0235	0.1652
		500	-0.0154	0.1402	-0.0491	0.2446	-0.0068	0.1446	-0.0325	0.1548	-0.0124	0.1420
		1000	-0.0037	0.1134	-0.0831	0.2005	0.0058	0.1145	-0.0177	0.1169	-0.0045	0.1102
	1.9	150	-0.0829	0.2306	-0.0947	0.4047	-0.0658	0.2143	-0.0774	0.2414	-0.0382	0.2115
		300	-0.0462	0.1725	-0.0611	0.3075	-0.0243	0.1642	-0.0492	0.1791	-0.0191	0.1658
		500	-0.0291	0.1438	-0.0554	0.2556	-0.0105	0.1346	-0.0343	0.1466	-0.0100	0.1380
		1000	-0.0170	0.1145	-0.0587	0.2153	0.0038	0.1065	-0.0191	0.1161	0.0004	0.1133

[a] The data generating process is $(1-L)^d X_t = (1-.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 35: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$ and $m = \lfloor n^{.65} \rfloor$

			fextLWF		fextLPWF(r=1)		ModLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	-0.1228	0.1846	-0.0804	0.2221	-0.0885	0.1692	-0.1120	0.1772	-0.1742	0.2286
		300	-0.0839	0.1326	-0.0446	0.1577	-0.0492	0.1236	-0.0745	0.1280	-0.1116	0.1546
		500	-0.0533	0.0995	-0.0297	0.1243	-0.0255	0.0951	-0.0508	0.1006	-0.0778	0.1175
		1000	-0.0326	0.0721	-0.0149	0.0888	-0.0082	0.0723	-0.0326	0.0737	-0.0482	0.0814
	.7	150	-0.1328	0.1844	-0.0953	0.2414	-0.1298	0.1847	-0.1006	0.1586	-0.1567	0.2129
		300	-0.0899	0.1334	-0.0589	0.1733	-0.0844	0.1303	-0.0688	0.1114	-0.1033	0.1445
		500	-0.0667	0.1070	-0.0414	0.1541	-0.0582	0.0982	-0.0515	0.0874	-0.0758	0.1090
		1000	-0.0391	0.0747	-0.0225	0.1122	-0.0363	0.0705	-0.0349	0.0640	-0.0495	0.0774
	.9	150	-0.1517	0.1986	-0.0910	0.2531	-0.1485	0.1963	-0.1262	0.1753	-0.1606	0.2184
		300	-0.0994	0.1386	-0.0475	0.1805	-0.0990	0.1381	-0.0890	0.1322	-0.1084	0.1548
		500	-0.0733	0.1079	-0.0352	0.1512	-0.0683	0.1048	-0.0648	0.1041	-0.0729	0.1142
		1000	-0.0454	0.0745	-0.0143	0.0992	-0.0453	0.0754	-0.0430	0.0756	-0.0442	0.0764
	1.1	150	-0.1567	0.2049	-0.0816	0.2727	-0.1582	0.2050	-0.1311	0.1876	-0.1374	0.1973
		300	-0.1040	0.1425	-0.0420	0.1749	-0.1024	0.1405	-0.0855	0.1302	-0.0902	0.1340
		500	-0.0760	0.1088	-0.0279	0.1333	-0.0739	0.1080	-0.0653	0.1012	-0.0646	0.1027
		1000	-0.0460	0.0743	-0.0164	0.0996	-0.0491	0.0762	-0.0417	0.0712	-0.0412	0.0724
	1.3	150	-0.1558	0.2033	-0.0712	0.2758	-0.1556	0.2035	-0.1337	0.1919	-0.1213	0.1788
		300	-0.1021	0.1410	-0.0273	0.1716	-0.1004	0.1383	-0.0871	0.1320	-0.0832	0.1275
		500	-0.0702	0.1048	-0.0144	0.1384	-0.0703	0.1049	-0.0614	0.0993	-0.0625	0.1001
		1000	-0.0421	0.0729	-0.0059	0.1014	-0.0443	0.0742	-0.0396	0.0709	-0.0386	0.0713
	1.5	150	-0.1436	0.1977	-0.0769	0.2709	-0.1515	0.2007	-0.1518	0.2216	-0.1170	0.1726
		300	-0.0870	0.1363	-0.0308	0.1590	-0.0940	0.1364	-0.0947	0.1551	-0.0798	0.1250
		500	-0.0558	0.1041	-0.0180	0.1253	-0.0659	0.1026	-0.0627	0.1137	-0.0576	0.0967
		1000	-0.0277	0.0757	-0.0062	0.0917	-0.0388	0.0718	-0.0324	0.0739	-0.0359	0.0691
	1.7	150	-0.1540	0.1972	-0.0851	0.2834	-0.1493	0.1989	-0.1338	0.1969	-0.1064	0.1686
		300	-0.0977	0.1345	-0.0350	0.1673	-0.0892	0.1334	-0.0838	0.1319	-0.0703	0.1181
		500	-0.0704	0.1056	-0.0200	0.1346	-0.0581	0.0999	-0.0608	0.1007	-0.0488	0.0910
		1000	-0.0432	0.0744	-0.0106	0.0994	-0.0318	0.0708	-0.0395	0.0720	-0.0297	0.0668
	1.9	150	-0.1776	0.2179	-0.0850	0.2793	-0.1638	0.2045	-0.1346	0.1917	-0.1013	0.1629
		300	-0.1097	0.1454	-0.0421	0.1684	-0.1022	0.1382	-0.0870	0.1293	-0.0641	0.1145
		500	-0.0801	0.1115	-0.0268	0.1336	-0.0674	0.1011	-0.0636	0.1013	-0.0437	0.0901
		1000	-0.0494	0.0766	-0.0139	0.0980	-0.0344	0.0679	-0.0411	0.0713	-0.0241	0.0651

[a] The data generating process is $(1-L)^d X_t = (1-.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 36: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$ and $m = \lfloor n^{.8} \rfloor$

			fextLWF		fextLPWF(r=1)		ModLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	-0.2747	0.2877	-0.1244	0.1818	-0.2434	0.2617	-0.2140	0.2308	-0.2492	0.2651
		300	-0.2249	0.2337	-0.0753	0.1194	-0.2005	0.2141	-0.1837	0.1937	-0.2104	0.2201
		500	-0.1994	0.2055	-0.0569	0.0925	-0.1734	0.1836	-0.1649	0.1718	-0.1834	0.1902
		1000	-0.1647	0.1687	-0.0381	0.0646	-0.1463	0.1533	-0.1416	0.1460	-0.1534	0.1577
	.7	150	-0.2740	0.2867	-0.1333	0.1826	-0.2836	0.2970	-0.1831	0.2089	-0.2304	0.2528
		300	-0.2279	0.2347	-0.0951	0.1357	-0.2313	0.2396	-0.1593	0.1759	-0.1914	0.2055
		500	-0.2019	0.2067	-0.0717	0.1070	-0.2047	0.2107	-0.1449	0.1564	-0.1682	0.1782
		1000	-0.1724	0.1759	-0.0480	0.0780	-0.1701	0.1740	-0.1237	0.1307	-0.1392	0.1456
	.9	150	-0.3037	0.3139	-0.1432	0.1989	-0.3069	0.3178	-0.1964	0.2084	-0.2248	0.2402
		300	-0.2492	0.2567	-0.0952	0.1355	-0.2501	0.2571	-0.1815	0.1890	-0.1964	0.2055
		500	-0.2166	0.2219	-0.0694	0.0998	-0.2190	0.2244	-0.1678	0.1736	-0.1776	0.1846
		1000	-0.1807	0.1840	-0.0482	0.0719	-0.1797	0.1831	-0.1453	0.1495	-0.1490	0.1536
	1.1	150	-0.3221	0.3324	-0.1359	0.1904	-0.3209	0.3318	-0.2138	0.2291	-0.2209	0.2387
		300	-0.2613	0.2685	-0.0944	0.1313	-0.2602	0.2673	-0.1888	0.1984	-0.1928	0.2028
		500	-0.2243	0.2295	-0.0729	0.1021	-0.2263	0.2314	-0.1711	0.1778	-0.1731	0.1799
		1000	-0.1860	0.1893	-0.0496	0.0723	-0.1848	0.1882	-0.1455	0.1494	-0.1461	0.1504
	1.3	150	-0.3261	0.3367	-0.1269	0.1861	-0.3321	0.3429	-0.2162	0.2328	-0.2153	0.2316
		300	-0.2632	0.2704	-0.0878	0.1281	-0.2661	0.2732	-0.1874	0.1973	-0.1862	0.1962
		500	-0.2292	0.2343	-0.0681	0.1011	-0.2287	0.2336	-0.1687	0.1757	-0.1686	0.1755
		1000	-0.1871	0.1905	-0.0473	0.0711	-0.1872	0.1906	-0.1442	0.1484	-0.1442	0.1484
	1.5	150	-0.3336	0.3450	-0.1233	0.1809	-0.3387	0.3495	-0.2202	0.2402	-0.2110	0.2282
		300	-0.2647	0.2723	-0.0764	0.1206	-0.2698	0.2771	-0.1899	0.2018	-0.1834	0.1934
		500	-0.2308	0.2361	-0.0577	0.0911	-0.2299	0.2356	-0.1685	0.1767	-0.1645	0.1715
		1000	-0.1872	0.1908	-0.0380	0.0648	-0.1881	0.1917	-0.1406	0.1458	-0.1410	0.1455
	1.7	150	-0.3272	0.3400	-0.1470	0.1906	-0.3440	0.3555	-0.2152	0.2332	-0.2011	0.2190
		300	-0.2576	0.2656	-0.0979	0.1381	-0.2697	0.2776	-0.1873	0.1975	-0.1769	0.1882
		500	-0.2243	0.2289	-0.0727	0.1073	-0.2295	0.2359	-0.1686	0.1755	-0.1584	0.1662
		1000	-0.1915	0.1941	-0.0450	0.0716	-0.1852	0.1895	-0.1437	0.1481	-0.1360	0.1408
	1.9	150	-0.3652	0.3714	-0.1575	0.2105	-0.3685	0.3782	-0.2155	0.2319	-0.1848	0.2046
		300	-0.2959	0.3020	-0.0978	0.1365	-0.2868	0.2934	-0.1887	0.1983	-0.1650	0.1770
		500	-0.2547	0.2595	-0.0718	0.1024	-0.2447	0.2496	-0.1704	0.1772	-0.1478	0.1567
		1000	-0.2049	0.2080	-0.0486	0.0711	-0.1961	0.1995	-0.1454	0.1496	-0.1257	0.1325

[a] The data generating process is $(1-L)^d X_t = (1 - .4L)\varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 37: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .4$ and $m = \lfloor n^{.5} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	0.0144	0.2134	-0.1394	0.4275	0.0210	0.2196	0.0195	0.2092	-0.0730	0.2522
		300	0.0147	0.1732	-0.0897	0.3033	0.0261	0.1752	0.0112	0.1670	-0.0577	0.2005
		500	0.0152	0.1458	-0.0579	0.2400	0.0223	0.1500	0.0108	0.1446	-0.0432	0.1684
		1000	0.0119	0.1171	-0.0449	0.1780	0.0247	0.1197	0.0090	0.1190	-0.0292	0.1298
	.7	150	0.0007	0.2048	-0.1452	0.4348	-0.0123	0.2222	0.0073	0.2017	-0.0705	0.2616
		300	-0.0028	0.1634	-0.0916	0.3168	-0.0034	0.1717	0.0005	0.1545	-0.0488	0.1974
		500	-0.0008	0.1378	-0.0693	0.2542	0.0004	0.1449	-0.0024	0.1264	-0.0384	0.1598
		1000	-0.0040	0.1148	-0.0491	0.1949	-0.0054	0.1127	0.0006	0.1020	-0.0280	0.1199
	.9	150	-0.0172	0.2165	-0.1253	0.4378	-0.0189	0.2183	-0.0088	0.2023	-0.0473	0.2555
		300	-0.0140	0.1691	-0.0978	0.3317	-0.0155	0.1666	-0.0083	0.1607	-0.0374	0.1961
		500	-0.0098	0.1378	-0.0662	0.2685	-0.0128	0.1422	-0.0129	0.1384	-0.0292	0.1619
		1000	-0.0134	0.1136	-0.0409	0.2005	-0.0098	0.1134	-0.0117	0.1136	-0.0207	0.1272
	1.1	150	-0.0121	0.2244	-0.1203	0.4453	-0.0243	0.2225	-0.0169	0.2169	-0.0298	0.2461
		300	-0.0137	0.1736	-0.0777	0.3374	-0.0213	0.1709	-0.0188	0.1677	-0.0227	0.1831
		500	-0.0146	0.1434	-0.0601	0.2706	-0.0110	0.1413	-0.0149	0.1440	-0.0172	0.1498
		1000	-0.0073	0.1133	-0.0350	0.2023	-0.0123	0.1113	-0.0137	0.1146	-0.0118	0.1148
	1.3	150	0.0003	0.2296	-0.0980	0.4464	-0.0175	0.2191	-0.0203	0.2320	-0.0126	0.2289
		300	-0.0039	0.1795	-0.0601	0.3292	-0.0135	0.1682	-0.0185	0.1839	-0.0091	0.1723
		500	-0.0043	0.1488	-0.0364	0.2658	-0.0104	0.1443	-0.0125	0.1505	-0.0082	0.1432
		1000	-0.0047	0.1175	-0.0322	0.1981	-0.0067	0.1128	-0.0087	0.1148	-0.0068	0.1131
	1.5	150	0.0006	0.2236	-0.0911	0.4256	-0.0082	0.2172	-0.0308	0.2424	0.0057	0.2143
		300	0.0097	0.1755	-0.0470	0.3088	-0.0035	0.1695	-0.0245	0.1926	0.0011	0.1669
		500	0.0078	0.1435	-0.0358	0.2500	-0.0028	0.1448	-0.0149	0.1615	-0.0016	0.1370
		1000	0.0112	0.1186	-0.0425	0.1823	0.0037	0.1135	-0.0138	0.1303	-0.0016	0.1117
	1.7	150	-0.0163	0.2138	-0.0908	0.4239	-0.0117	0.2029	-0.0191	0.2361	0.0135	0.2134
		300	-0.0060	0.1652	-0.0551	0.3005	0.0063	0.1673	-0.0124	0.1774	0.0070	0.1644
		500	-0.0034	0.1394	-0.0422	0.2395	0.0100	0.1401	-0.0121	0.1478	0.0072	0.1388
		1000	-0.0028	0.1124	-0.0747	0.2021	0.0149	0.1139	-0.0065	0.1141	0.0077	0.1098
	1.9	150	-0.0217	0.2171	-0.0786	0.4116	-0.0575	0.1759	-0.0102	0.2277	0.0162	0.2056
		300	-0.0166	0.1669	-0.0635	0.3057	-0.0278	0.1320	-0.0182	0.1750	0.0085	0.1576
		500	-0.0121	0.1437	-0.0469	0.2529	-0.0161	0.1129	-0.0127	0.1440	0.0099	0.1327
		1000	-0.0092	0.1116	-0.0547	0.2154	-0.0031	0.0938	-0.0091	0.1133	0.0049	0.1089

[a] The data generating process is $(1-L)^d X_t = (1+.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 38: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .4$ and $m = \lfloor n^{.65} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	0.0329	0.1334	-0.0473	0.2269	0.0400	0.1394	0.0578	0.1427	0.0135	0.1480
		300	0.0180	0.1000	-0.0176	0.1541	0.0277	0.1028	0.0393	0.1076	0.0104	0.1076
		500	0.0126	0.0810	-0.0146	0.1278	0.0245	0.0855	0.0308	0.0895	0.0070	0.0893
		1000	0.0083	0.0620	-0.0015	0.0910	0.0196	0.0663	0.0221	0.0713	0.0027	0.0690
	.7	150	0.0221	0.1311	-0.0501	0.2322	0.0198	0.1302	0.0418	0.1301	0.0127	0.1447
		300	0.0124	0.0984	-0.0353	0.1647	0.0167	0.1003	0.0237	0.0930	0.0017	0.1007
		500	0.0085	0.0802	-0.0229	0.1440	0.0088	0.0784	0.0141	0.0744	0.0007	0.0818
		1000	0.0073	0.0612	-0.0140	0.1038	0.0066	0.0602	0.0083	0.0564	-0.0019	0.0619
	.9	150	0.0143	0.1285	-0.0530	0.2464	0.0105	0.1281	0.0386	0.1371	0.0266	0.1442
		300	0.0074	0.0943	-0.0290	0.1701	0.0038	0.0959	0.0174	0.1016	0.0135	0.1050
		500	0.0012	0.0790	-0.0259	0.1422	0.0027	0.0786	0.0113	0.0794	0.0103	0.0837
		1000	0.0024	0.0596	-0.0131	0.0984	-0.0001	0.0586	0.0050	0.0600	0.0059	0.0605
	1.1	150	0.0068	0.1312	-0.0501	0.2659	0.0024	0.1293	0.0404	0.1388	0.0374	0.1393
		300	0.0021	0.0934	-0.0274	0.1737	0.0036	0.0954	0.0187	0.0994	0.0211	0.0979
		500	0.0007	0.0789	-0.0226	0.1347	0.0011	0.0770	0.0124	0.0778	0.0128	0.0797
		1000	-0.0001	0.0592	-0.0135	0.0981	0.0019	0.0587	0.0067	0.0596	0.0067	0.0610
	1.3	150	0.0123	0.1356	-0.0334	0.2639	0.0064	0.1298	0.0412	0.1413	0.0436	0.1385
		300	0.0072	0.0993	-0.0081	0.1752	0.0044	0.0955	0.0229	0.1021	0.0212	0.1003
		500	0.0053	0.0805	-0.0078	0.1416	0.0026	0.0784	0.0127	0.0789	0.0178	0.0800
		1000	0.0033	0.0600	-0.0010	0.1043	0.0029	0.0592	0.0084	0.0595	0.0095	0.0592
	1.5	150	0.0170	0.1305	-0.0315	0.2636	0.0018	0.1292	0.0352	0.1492	0.0462	0.1362
		300	0.0163	0.0995	-0.0081	0.1557	0.0055	0.0972	0.0229	0.1073	0.0293	0.0980
		500	0.0140	0.0819	-0.0002	0.1292	0.0082	0.0783	0.0128	0.0850	0.0184	0.0808
		1000	0.0150	0.0629	0.0050	0.0926	0.0077	0.0606	0.0112	0.0618	0.0124	0.0603
	1.7	150	-0.0053	0.1289	-0.0470	0.2576	-0.0062	0.1316	0.0363	0.1415	0.0522	0.1355
		300	0.0002	0.0947	-0.0210	0.1567	0.0062	0.0972	0.0227	0.1019	0.0304	0.0980
		500	0.0038	0.0771	-0.0126	0.1303	0.0122	0.0806	0.0150	0.0807	0.0255	0.0819
		1000	0.0033	0.0592	-0.0073	0.1006	0.0136	0.0642	0.0080	0.0599	0.0170	0.0630
	1.9	150	-0.0138	0.1277	-0.0530	0.2618	-0.0646	0.1343	0.0358	0.1385	0.0475	0.1305
		300	-0.0093	0.0966	-0.0240	0.1648	-0.0276	0.0915	0.0196	0.0984	0.0317	0.0970
		500	-0.0065	0.0785	-0.0201	0.1306	-0.0135	0.0736	0.0140	0.0791	0.0240	0.0768
		1000	-0.0015	0.0579	-0.0121	0.0983	0.0020	0.0553	0.0078	0.0593	0.0171	0.0606

[a] The data generating process is $(1-L)^d X_t = (1+.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 39: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .4$ and $m = \lfloor n^{.8} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	0.1354	0.1579	-0.0002	0.1301	0.1313	0.1537	0.2447	0.2599	0.2365	0.2543
		300	0.0937	0.1121	0.0002	0.0897	0.0961	0.1131	0.1629	0.1730	0.1555	0.1670
		500	0.0749	0.0896	0.0075	0.0710	0.0771	0.0902	0.1296	0.1371	0.1255	0.1337
		1000	0.0542	0.0656	0.0051	0.0510	0.0566	0.0667	0.0961	0.1021	0.0915	0.0980
	.7	150	0.1160	0.1423	-0.0118	0.1277	0.1108	0.1366	0.2401	0.2565	0.2415	0.2580
		300	0.0799	0.0988	-0.0069	0.0932	0.0775	0.0969	0.1584	0.1701	0.1586	0.1706
		500	0.0645	0.0788	-0.0054	0.0731	0.0624	0.0767	0.1247	0.1338	0.1201	0.1296
		1000	0.0460	0.0568	-0.0011	0.0527	0.0451	0.0560	0.0870	0.0935	0.0856	0.0927
	.9	150	0.0974	0.1248	-0.0229	0.1310	0.0940	0.1236	0.2390	0.2551	0.2395	0.2550
		300	0.0675	0.0890	-0.0132	0.0939	0.0655	0.0866	0.1576	0.1693	0.1571	0.1690
		500	0.0543	0.0702	-0.0099	0.0737	0.0532	0.0699	0.1226	0.1316	0.1225	0.1316
		1000	0.0392	0.0511	-0.0040	0.0521	0.0387	0.0506	0.0871	0.0936	0.0865	0.0932
	1.1	150	0.0828	0.1142	-0.0230	0.1349	0.0782	0.1110	0.2406	0.2567	0.2386	0.2548
		300	0.0582	0.0811	-0.0135	0.0945	0.0568	0.0811	0.1571	0.1684	0.1564	0.1684
		500	0.0468	0.0648	-0.0106	0.0724	0.0448	0.0639	0.1218	0.1310	0.1213	0.1305
		1000	0.0345	0.0475	-0.0045	0.0519	0.0338	0.0473	0.0860	0.0926	0.0862	0.0927
	1.3	150	0.0754	0.1098	-0.0137	0.1365	0.0640	0.1043	0.2414	0.2573	0.2425	0.2576
		300	0.0526	0.0786	-0.0093	0.0962	0.0488	0.0755	0.1599	0.1706	0.1589	0.1703
		500	0.0426	0.0618	-0.0039	0.0735	0.0395	0.0610	0.1240	0.1332	0.1228	0.1316
		1000	0.0296	0.0447	-0.0004	0.0522	0.0307	0.0457	0.0875	0.0941	0.0874	0.0939
	1.5	150	0.0509	0.0906	-0.0128	0.1207	0.0329	0.0940	0.2431	0.2587	0.2413	0.2570
		300	0.0422	0.0706	-0.0050	0.0847	0.0329	0.0685	0.1610	0.1727	0.1618	0.1730
		500	0.0364	0.0579	-0.0009	0.0671	0.0308	0.0565	0.1263	0.1350	0.1257	0.1344
		1000	0.0279	0.0441	0.0033	0.0502	0.0262	0.0430	0.0894	0.0961	0.0892	0.0958
	1.7	150	0.0182	0.0885	-0.0424	0.1450	-0.0247	0.1037	0.2423	0.2581	0.2409	0.2564
		300	0.0234	0.0630	-0.0189	0.1026	0.0019	0.0668	0.1590	0.1705	0.1643	0.1752
		500	0.0220	0.0511	-0.0093	0.0783	0.0109	0.0514	0.1229	0.1317	0.1289	0.1374
		1000	0.0181	0.0380	-0.0037	0.0537	0.0161	0.0388	0.0870	0.0937	0.0927	0.0991
	1.9	150	-0.0034	0.0856	-0.0371	0.1479	-0.1520	0.2067	0.2418	0.2575	0.2207	0.2352
		300	0.0077	0.0605	-0.0126	0.0955	-0.0994	0.1425	0.1569	0.1684	0.1514	0.1617
		500	0.0106	0.0471	-0.0099	0.0724	-0.0754	0.1103	0.1224	0.1311	0.1212	0.1292
		1000	0.0108	0.0350	-0.0042	0.0514	-0.0500	0.0775	0.0871	0.0937	0.0895	0.0954

[a] The data generating process is $(1-L)^d X_t = (1 + .4L)\varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 40: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$ and $m = \lfloor n^{.5} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	0.0221	0.2123	-0.1323	0.4171	0.0235	0.2199	0.0231	0.2114	-0.0694	0.2503
		300	0.0152	0.1716	-0.0860	0.3036	0.0230	0.1768	0.0174	0.1697	-0.0584	0.1973
		500	0.0122	0.1472	-0.0584	0.2460	0.0241	0.1487	0.0130	0.1468	-0.0437	0.1683
		1000	0.0126	0.1184	-0.0437	0.1791	0.0214	0.1204	0.0099	0.1168	-0.0299	0.1313
	.7	150	0.0017	0.2055	-0.1298	0.4208	-0.0039	0.2158	0.0138	0.2014	-0.0638	0.2585
		300	-0.0061	0.1643	-0.0930	0.3155	-0.0028	0.1700	0.0021	0.1514	-0.0532	0.1974
		500	-0.0005	0.1384	-0.0734	0.2584	-0.0029	0.1432	-0.0016	0.1255	-0.0377	0.1587
		1000	-0.0016	0.1128	-0.0423	0.1919	-0.0034	0.1132	0.0018	0.1022	-0.0277	0.1220
	.9	150	-0.0137	0.2141	-0.1340	0.4371	-0.0175	0.2221	-0.0028	0.2032	-0.0497	0.2561
		300	-0.0169	0.1695	-0.0841	0.3278	-0.0127	0.1690	-0.0136	0.1645	-0.0346	0.1930
		500	-0.0125	0.1438	-0.0705	0.2668	-0.0117	0.1434	-0.0075	0.1384	-0.0276	0.1592
		1000	-0.0087	0.1107	-0.0353	0.2015	-0.0085	0.1152	-0.0117	0.1120	-0.0205	0.1272
	1.1	150	-0.0170	0.2290	-0.1226	0.4548	-0.0214	0.2189	-0.0126	0.2180	-0.0299	0.2470
		300	-0.0170	0.1676	-0.0753	0.3369	-0.0179	0.1696	-0.0161	0.1697	-0.0172	0.1833
		500	-0.0096	0.1444	0.2668	0.2678	-0.0123	0.1399	-0.0108	0.1411	-0.0196	0.1480
		1000	-0.0106	0.1118	-0.0392	0.1993	-0.0131	0.1126	-0.0111	0.1133	-0.0124	0.1121
	1.3	150	-0.0057	0.2285	-0.0953	0.4462	-0.0202	0.2202	-0.0146	0.2341	-0.0064	0.2232
		300	-0.0060	0.1813	-0.0550	0.3277	-0.0116	0.1720	-0.0150	0.1796	-0.0104	0.1746
		500	-0.0057	0.1474	-0.0402	0.2673	-0.0053	0.1428	-0.0113	0.1470	-0.0072	0.1406
		1000	-0.0047	0.1198	-0.0336	0.2031	-0.0016	0.1141	-0.0053	0.1171	-0.0044	0.1113
	1.5	150	-0.0005	0.2192	-0.0788	0.4162	-0.0123	0.2248	-0.0262	0.2479	0.0045	0.2170
		300	0.0074	0.1702	-0.0529	0.3136	-0.0076	0.1703	-0.0212	0.1941	-0.0022	0.1665
		500	0.0111	0.1499	-0.0334	0.2487	0.0000	0.1435	-0.0189	0.1600	0.0006	0.1377
		1000	0.0119	0.1206	-0.0444	0.1828	0.0021	0.1132	-0.0093	0.1277	-0.0024	0.1095
	1.7	150	-0.0063	0.2090	-0.0824	0.4210	0.0032	0.2147	-0.0163	0.2340	0.0137	0.2128
		300	-0.0050	0.1670	-0.0499	0.3035	0.0078	0.1709	-0.0191	0.1789	0.0032	0.1675
		500	-0.0030	0.1371	-0.0373	0.2455	0.0110	0.1418	-0.0148	0.1493	0.0049	0.1386
		1000	0.0004	0.1122	-0.0789	0.2026	0.0078	0.1163	-0.0095	0.1150	0.0031	0.1103
	1.9	150	-0.0258	0.2185	-0.0694	0.4175	-0.0233	0.2105	-0.0116	0.2264	0.0192	0.2025
		300	-0.0182	0.1687	-0.0581	0.3020	-0.0061	0.1588	-0.0185	0.1705	0.0110	0.1584
		500	-0.0132	0.1410	-0.0474	0.2524	-0.0017	0.1334	-0.0105	0.1399	0.0051	0.1333
		1000	-0.0097	0.1138	-0.0598	0.2172	0.0092	0.1057	-0.0098	0.1124	0.0033	0.1083

[a] The data generating process is $(1-L)^d X_t = (1 + .8L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 41: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$ and $m = \lfloor n^{.65} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	0.0376	0.1358	-0.0513	0.2279	0.0443	0.1389	0.0688	0.1448	0.0205	0.1490
		300	0.0252	0.1006	-0.0209	0.1579	0.0314	0.1032	0.0483	0.1116	0.0165	0.1089
		500	0.0169	0.0819	-0.0133	0.1274	0.0269	0.0852	0.0344	0.0911	0.0098	0.0895
		1000	0.0132	0.0625	-0.0026	0.0900	0.0218	0.0674	0.0247	0.0714	0.0070	0.0686
	.7	150	0.0278	0.1340	-0.0507	0.2305	0.0291	0.1330	0.0539	0.1364	0.0250	0.1439
		300	0.0159	0.0979	-0.0307	0.1639	0.0178	0.0976	0.0261	0.0951	0.0074	0.1029
		500	0.0111	0.0808	-0.0236	0.1442	0.0126	0.0787	0.0177	0.0757	0.0031	0.0822
		1000	0.0086	0.0611	-0.0136	0.1034	0.0074	0.0592	0.0130	0.0581	0.0009	0.0612
	.9	150	0.0202	0.1328	-0.0562	0.2566	0.0194	0.1294	0.0446	0.1411	0.0381	0.1458
		300	0.0102	0.0965	-0.0321	0.1717	0.0088	0.0970	0.0244	0.1019	0.0215	0.1038
		500	0.0064	0.0773	-0.0210	0.1374	0.0077	0.0775	0.0164	0.0824	0.0124	0.0830
		1000	0.0023	0.0595	-0.0136	0.0995	0.0030	0.0584	0.0073	0.0608	0.0074	0.0607
	1.1	150	0.0150	0.1318	-0.0409	0.2603	0.0119	0.1310	0.0466	0.1381	0.0466	0.1422
		300	0.0055	0.0958	-0.0270	0.1729	0.0076	0.0944	0.0249	0.0987	0.0263	0.1001
		500	0.0065	0.0783	-0.0213	0.1339	0.0036	0.0765	0.0155	0.0813	0.0151	0.0793
		1000	0.0007	0.0603	-0.0121	0.0996	0.0037	0.0581	0.0087	0.0587	0.0097	0.0610
	1.3	150	0.0213	0.1346	-0.0269	0.2606	0.0148	0.1301	0.0511	0.1447	0.0513	0.1415
		300	0.0094	0.0986	-0.0079	0.1758	0.0098	0.0966	0.0266	0.1040	0.0295	0.0998
		500	0.0065	0.0810	-0.0062	0.1448	0.0064	0.0788	0.0164	0.0790	0.0200	0.0794
		1000	0.0031	0.0605	-0.0034	0.1059	0.0051	0.0609	0.0108	0.0603	0.0119	0.0599
	1.5	150	0.0206	0.1311	-0.0305	0.2570	0.0080	0.1320	0.0436	0.1518	0.0557	0.1408
		300	0.0198	0.0982	-0.0029	0.1586	0.0121	0.0987	0.0231	0.1061	0.0320	0.1007
		500	0.0183	0.0825	0.0011	0.1225	0.0102	0.0796	0.0180	0.0836	0.0225	0.0802
		1000	0.0159	0.0643	0.0019	0.0928	0.0100	0.0613	0.0126	0.0616	0.0147	0.0615
	1.7	150	0.0015	0.1274	-0.0447	0.2582	-0.0017	0.1288	0.0476	0.1419	0.0602	0.1390
		300	0.0050	0.0958	-0.0195	0.1573	0.0101	0.0971	0.0276	0.1017	0.0386	0.1005
		500	0.0046	0.0768	-0.0124	0.1317	0.0132	0.0810	0.0176	0.0801	0.0264	0.0815
		1000	0.0053	0.0607	-0.0072	0.1007	0.0172	0.0653	0.0104	0.0608	0.0191	0.0625
	1.9	150	-0.0089	0.1296	-0.0510	0.2650	-0.0527	0.1405	0.0440	0.1409	0.0542	0.1327
		300	-0.0023	0.0953	-0.0259	0.1626	-0.0205	0.0954	0.0241	0.0993	0.0396	0.0989
		500	-0.0020	0.0780	-0.0184	0.1313	-0.0080	0.0747	0.0170	0.0795	0.0264	0.0795
		1000	0.0004	0.0591	-0.0111	0.0976	0.0047	0.0563	0.0081	0.0602	0.0209	0.0617

[a] The data generating process is $(1-L)^d X_t = (1+.8L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 42: Monte Carlo Results of the Local Whittle Fourier Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$ and $m = \lfloor n^{.8} \rfloor$

			fextLWF		fextLPWF(r=1)		modLWF		feLWF		dtr-feLWF	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	0.1963	0.2126	-0.0165	0.1329	0.1903	0.2074	0.3122	0.3258	0.3098	0.3245
		300	0.1299	0.1433	-0.0008	0.0907	0.1283	0.1417	0.2000	0.2089	0.1970	0.2071
		500	0.1014	0.1125	0.0041	0.0696	0.1027	0.1129	0.1562	0.1626	0.1504	0.1574
		1000	0.0728	0.0816	0.0033	0.0503	0.0744	0.0820	0.1126	0.1176	0.1081	0.1133
	.7	150	0.1771	0.1959	-0.0304	0.1278	0.1703	0.1881	0.3116	0.3250	0.3093	0.3227
		300	0.1152	0.1293	-0.0172	0.0947	0.1131	0.1273	0.1975	0.2076	0.1961	0.2057
		500	0.0896	0.1005	-0.0085	0.0727	0.0888	0.0998	0.1512	0.1589	0.1501	0.1579
		1000	0.0635	0.0717	-0.0041	0.0538	0.0631	0.0712	0.1063	0.1119	0.1049	0.1106
	.9	150	0.1588	0.1775	-0.0404	0.1382	0.1548	0.1747	0.3103	0.3235	0.3094	0.3225
		300	0.1029	0.1184	-0.0219	0.0932	0.1014	0.1165	0.1964	0.2065	0.1966	0.2062
		500	0.0789	0.0911	-0.0128	0.0725	0.0787	0.0906	0.1499	0.1573	0.1501	0.1577
		1000	0.0574	0.0663	-0.0060	0.0532	0.0565	0.0651	0.1048	0.1106	0.1047	0.1105
	1.1	150	0.1440	0.1648	-0.0410	0.1391	0.1377	0.1594	0.3089	0.3221	0.3082	0.3218
		300	0.0938	0.1096	-0.0199	0.0933	0.0919	0.1084	0.1982	0.2075	0.1973	0.2070
		500	0.0712	0.0849	-0.0123	0.0729	0.0708	0.0844	0.1497	0.1575	0.1504	0.1581
		1000	0.0509	0.0608	-0.0060	0.0522	0.0503	0.0602	0.1053	0.1108	0.1051	0.1108
	1.3	150	0.1360	0.1577	-0.0329	0.1413	0.1240	0.1493	0.3130	0.3263	0.3101	0.3237
		300	0.0886	0.1069	-0.0112	0.0968	0.0826	0.1007	0.1979	0.2074	0.1984	0.2081
		500	0.0680	0.0824	-0.0080	0.0740	0.0656	0.0801	0.1523	0.1599	0.1520	0.1595
		1000	0.0476	0.0581	-0.0048	0.0531	0.0467	0.0577	0.1060	0.1114	0.1053	0.1110
	1.5	150	0.1043	0.1313	-0.0258	0.1221	0.0825	0.1242	0.3125	0.3259	0.3098	0.3227
		300	0.0748	0.0952	-0.0077	0.0877	0.0665	0.0902	0.2015	0.2113	0.1991	0.2091
		500	0.0607	0.0765	-0.0043	0.0691	0.0556	0.0728	0.1538	0.1613	0.1531	0.1606
		1000	0.0452	0.0569	0.0002	0.0509	0.0425	0.0547	0.1082	0.1139	0.1069	0.1126
	1.7	150	0.0740	0.1170	-0.0529	0.1450	0.0139	0.1126	0.3093	0.3229	0.3038	0.3164
		300	0.0579	0.0826	-0.0234	0.1019	0.0268	0.0777	0.1981	0.2077	0.1985	0.2078
		500	0.0484	0.0663	-0.0126	0.0784	0.0305	0.0600	0.1509	0.1584	0.1540	0.1615
		1000	0.0351	0.0485	-0.0051	0.0529	0.0303	0.0468	0.1059	0.1116	0.1108	0.1164
	1.9	150	0.0504	0.1027	-0.0498	0.1545	-0.1309	0.2014	0.3081	0.3216	0.2722	0.2863
		300	0.0418	0.0736	-0.0232	0.1009	-0.0844	0.1396	0.1951	0.2049	0.1819	0.1914
		500	0.0357	0.0581	-0.0163	0.0741	-0.0647	0.1090	0.1497	0.1575	0.1438	0.1509
		1000	0.0280	0.0434	-0.0071	0.0522	-0.0421	0.0755	0.1060	0.1115	0.1060	0.1110

[a] The data generating process is $(1-L)^d X_t = (1+.8L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The m is the number of the frequency ordinates involved in the estimation.

Table 43: Monte Carlo Results of the Log-Regression Wavelet Estimation for ARFIMA$(0, d, 0)$ Model

		K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	-0.1155	0.2350	-0.1712	0.4358	-0.0677	0.2132	-0.0765	0.5059
	300	-0.0726	0.1333	-0.0949	0.2241	-0.0508	0.1195	-0.0493	0.2102
	500	-0.0807	0.1252	-0.1084	0.2013	-0.0431	0.0837	-0.0408	0.1327
	1000	-0.0513	0.0753	-0.0656	0.1169	-0.0327	0.0556	-0.0270	0.0776
.7	150	-0.1194	0.2365	-0.1640	0.4263	-0.0811	0.2183	-0.0731	0.4958
	300	-0.0794	0.1373	-0.0965	0.2293	-0.0571	0.1243	-0.0450	0.2097
	500	-0.0817	0.1265	-0.1079	0.2003	-0.0494	0.0909	-0.0430	0.1373
	1000	-0.0546	0.0784	-0.0641	0.1171	-0.0357	0.0582	-0.0288	0.0803
.9	150	-0.1252	0.2394	-0.1699	0.4314	-0.0890	0.2253	-0.0879	0.5013
	300	-0.0803	0.1404	-0.1015	0.2323	-0.0584	0.1298	-0.0548	0.2151
	500	-0.0820	0.1271	-0.1100	0.2020	-0.0502	0.0934	-0.0462	0.1389
	1000	-0.0569	0.0815	-0.0645	0.1166	-0.0381	0.0624	-0.0312	0.0842
1.1	150	-0.1248	0.2423	-0.1802	0.4545	-0.0878	0.2264	-0.0848	0.4865
	300	-0.0860	0.1495	-0.1002	0.2382	-0.0598	0.1297	-0.0561	0.2181
	500	-0.0878	0.1333	-0.1094	0.2038	-0.0532	0.0962	-0.0505	0.1425
	1000	-0.0594	0.0864	-0.0665	0.1191	-0.0394	0.0650	-0.0307	0.0854
1.3	150	-0.1310	0.2477	-0.1753	0.4393	-0.0878	0.2235	-0.0815	0.4819
	300	-0.0837	0.1500	-0.1031	0.2355	-0.0600	0.1311	-0.0551	0.2136
	500	-0.0864	0.1358	-0.1097	0.2041	-0.0521	0.0971	-0.0461	0.1414
	1000	-0.0569	0.0845	-0.0666	0.1224	-0.0402	0.0663	-0.0300	0.0871
1.5	150	-0.1245	0.2506	-0.1740	0.4384	-0.0783	0.2182	-0.0822	0.4621
	300	-0.0821	0.1484	-0.1036	0.2366	-0.0591	0.1311	-0.0541	0.2123
	500	-0.0799	0.1293	-0.1105	0.2063	-0.0482	0.0947	-0.0457	0.1423
	1000	-0.0571	0.0853	-0.0684	0.1246	-0.0381	0.0647	-0.0301	0.0884
1.7	150	-0.1139	0.2359	-0.1757	0.4233	-0.0754	0.2089	-0.0851	0.4292
	300	-0.0752	0.1469	-0.1002	0.2323	-0.0553	0.1281	-0.0519	0.2004
	500	-0.0805	0.1311	-0.1055	0.2040	-0.0450	0.0954	-0.0441	0.1377
	1000	-0.0537	0.0844	-0.0665	0.1262	-0.0349	0.0648	-0.0287	0.0880
1.9	150	-0.1093	0.2317	-0.1656	0.4204	-0.0652	0.1923	-0.0754	0.3794
	300	-0.0669	0.1405	-0.0902	0.2246	-0.0482	0.1222	-0.0485	0.1874
	500	-0.0783	0.1328	-0.1108	0.2131	-0.0408	0.0905	-0.0355	0.1312
	1000	-0.0487	0.0838	-0.0652	0.1264	-0.0310	0.0635	-0.0267	0.0863

[a] The data generation is $(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 44: Monte Carlo Results of the Log-Regression Wavelet Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	0.0898	0.2185	-0.0502	0.3995	0.1849	0.2745	0.0728	0.4926
		300	0.1042	0.1547	-0.0110	0.2025	0.1540	0.1891	0.0570	0.2125
		500	0.0775	0.1250	-0.0354	0.1705	0.1362	0.1544	0.0417	0.1314
		1000	0.0967	0.1113	-0.0005	0.0941	0.1266	0.1348	0.0437	0.0862
	.7	150	0.0708	0.2164	-0.0740	0.4126	0.1548	0.2577	0.0563	0.4944
		300	0.0888	0.1460	-0.0191	0.2043	0.1389	0.1785	0.0459	0.2131
		500	0.0653	0.1163	-0.0469	0.1792	0.1179	0.1409	0.0378	0.1330
		1000	0.0812	0.0999	-0.0064	0.0946	0.1134	0.1230	0.0362	0.0840
	.9	150	0.0528	0.2126	-0.0904	0.4132	0.1410	0.2511	0.0380	0.4870
		300	0.0681	0.1355	-0.0289	0.2112	0.1193	0.1643	0.0294	0.2117
		500	0.0513	0.1117	-0.0485	0.1782	0.1027	0.1291	0.0243	0.1328
		1000	0.0706	0.0921	-0.0159	0.1030	0.0983	0.1102	0.0273	0.0839
	1.1	150	0.0291	0.2124	-0.0959	0.4149	0.1130	0.2380	0.0264	0.4741
		300	0.0560	0.1311	-0.0387	0.2165	0.1024	0.1552	0.0181	0.2134
		500	0.0367	0.1072	-0.0551	0.1869	0.0864	0.1185	0.0138	0.1372
		1000	0.0563	0.0833	-0.0215	0.1002	0.0845	0.0989	0.0208	0.0823
	1.3	150	0.0218	0.2080	-0.1127	0.4218	0.0945	0.2215	0.0031	0.4607
		300	0.0396	0.1275	-0.0473	0.2119	0.0839	0.1430	0.0172	0.2089
		500	0.0274	0.1050	-0.0639	0.1869	0.0740	0.1100	0.0054	0.1353
		1000	0.0421	0.0758	-0.0268	0.1042	0.0716	0.0889	0.0136	0.0822
	1.5	150	-0.0029	0.2086	-0.1211	0.4170	0.0761	0.2082	-0.0010	0.4333
		300	0.0264	0.1260	-0.0537	0.2202	0.0657	0.1321	0.0046	0.2015
		500	0.0119	0.1038	-0.0726	0.1934	0.0596	0.1004	0.0008	0.1345
		1000	0.0337	0.0712	-0.0324	0.1077	0.0589	0.0793	0.0082	0.0832
	1.7	150	-0.0154	0.2077	-0.1156	0.4022	0.0578	0.1928	-0.0184	0.4093
		300	0.0133	0.1237	-0.0572	0.2158	0.0494	0.1229	-0.0088	0.1887
		500	0.0016	0.1050	-0.0744	0.1937	0.0457	0.0922	-0.0078	0.1285
		1000	0.0193	0.0665	-0.0356	0.1112	0.0465	0.0701	0.0026	0.0801
	1.9	150	-0.0298	0.2033	-0.1343	0.4015	0.0367	0.1725	-0.0217	0.3555
		300	-0.0008	0.1218	-0.0635	0.2140	0.0336	0.1100	-0.0155	0.1733
		500	-0.0127	0.1078	-0.0789	0.1972	0.0299	0.0848	-0.0095	0.1248
		1000	0.0067	0.0677	-0.0420	0.1158	0.0330	0.0626	-0.0025	0.0821

[a] The data generation is $(1 + .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 45: Monte Carlo Results of the Log-Regression Wavelet Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .4$

ϕ	d	n	K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
			bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	-0.2108	0.2928	-0.2142	0.4591	-0.1920	0.2776	-0.1307	0.5062
		300	-0.1593	0.1952	-0.1221	0.2349	-0.1512	0.1852	-0.0817	0.2229
		500	-0.1527	0.1793	-0.1359	0.2204	-0.1275	0.1472	-0.0664	0.1401
		1000	-0.1212	0.1338	-0.0859	0.1286	-0.1063	0.1154	-0.0521	0.0887
	.7	150	-0.2121	0.2948	-0.2044	0.4430	-0.1950	0.2843	-0.1278	0.5056
		300	-0.1563	0.1936	-0.1173	0.2356	-0.1483	0.1841	-0.0800	0.2240
		500	-0.1506	0.1800	-0.1308	0.2131	-0.1247	0.1462	-0.0691	0.1457
		1000	-0.1174	0.1302	-0.0832	0.1281	-0.1044	0.1145	-0.0477	0.0882
	.9	150	-0.2104	0.2957	-0.1962	0.4532	-0.1898	0.2806	-0.1266	0.5075
		300	-0.1549	0.1942	-0.1188	0.2367	-0.1413	0.1818	-0.0777	0.2250
		500	-0.1456	0.1748	-0.1296	0.2168	-0.1192	0.1422	-0.0662	0.1460
		1000	-0.1144	0.1291	-0.0834	0.1292	-0.1010	0.1122	-0.0453	0.0905
	1.1	150	-0.1983	0.2887	-0.1888	0.4419	-0.1789	0.2731	-0.1171	0.5004
		300	-0.1460	0.1901	-0.1113	0.2380	-0.1334	0.1781	-0.0738	0.2226
		500	-0.1408	0.1726	-0.1283	0.2200	-0.1137	0.1393	-0.0640	0.1484
		1000	-0.1072	0.1230	-0.0786	0.1271	-0.0941	0.1070	-0.0435	0.0911
	1.3	150	-0.1886	0.2798	-0.1909	0.4487	-0.1709	0.2678	-0.1143	0.4969
		300	-0.1365	0.1819	-0.1113	0.2373	-0.1232	0.1713	-0.0729	0.2190
		500	-0.1332	0.1694	-0.1201	0.2101	-0.1062	0.1345	-0.0592	0.1477
		1000	-0.1005	0.1183	-0.0779	0.1276	-0.0879	0.1023	-0.0427	0.0918
	1.5	150	-0.1769	0.2783	-0.1999	0.4524	-0.1501	0.2522	-0.0960	0.4573
		300	-0.1279	0.1781	-0.1053	0.2357	-0.1100	0.1610	-0.0672	0.2148
		500	-0.1250	0.1638	-0.1164	0.2136	-0.0956	0.1269	-0.0570	0.1470
		1000	-0.0922	0.1123	-0.0730	0.1260	-0.0786	0.0953	-0.0390	0.0916
	1.7	150	-0.1626	0.2665	-0.1740	0.4282	-0.1341	0.2386	-0.0993	0.4309
		300	-0.1117	0.1681	-0.1024	0.2357	-0.0966	0.1517	-0.0617	0.2027
		500	-0.1111	0.1530	-0.1174	0.2125	-0.0810	0.1161	-0.0487	0.1401
		1000	-0.0831	0.1053	-0.0694	0.1262	-0.0668	0.0864	-0.0371	0.0914
	1.9	150	-0.1433	0.2548	-0.1685	0.4185	-0.1082	0.2120	-0.0909	0.3864
		300	-0.1000	0.1605	-0.1009	0.2312	-0.0786	0.1361	-0.0546	0.1869
		500	-0.1005	0.1464	-0.1102	0.2084	-0.0678	0.1078	-0.0470	0.1348
		1000	-0.0708	0.0974	-0.0666	0.1272	-0.0557	0.0788	-0.0322	0.0874

[a] The data generation is $(1 - .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 46: Monte Carlo Results of the Log-Regression Wavelet Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = .8$

ϕ	d	n	K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
			bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	-0.2452	0.3156	-0.2331	0.4766	-0.2341	0.3080	-0.1547	0.5185
		300	-0.1911	0.2215	-0.1438	0.2510	-0.1836	0.2125	-0.1065	0.2297
		500	-0.1805	0.2035	-0.1502	0.2246	-0.1579	0.1738	-0.0890	0.1507
		1000	-0.1453	0.1555	-0.0977	0.1357	-0.1350	0.1425	-0.0662	0.0983
	.7	150	-0.2461	0.3208	-0.2185	0.4657	-0.2355	0.3136	-0.1358	0.5001
		300	-0.1848	0.2171	-0.1339	0.2427	-0.1811	0.2119	-0.0960	0.2269
		500	-0.1742	0.1993	-0.1416	0.2197	-0.1533	0.1706	-0.0766	0.1511
		1000	-0.1406	0.1515	-0.0914	0.1334	-0.1293	0.1375	-0.0584	0.0945
	.9	150	-0.2332	0.3095	-0.2169	0.4608	-0.2273	0.3067	-0.1294	0.5003
		300	-0.1749	0.2091	-0.1256	0.2404	-0.1725	0.2076	-0.0816	0.2247
		500	-0.1667	0.1935	-0.1320	0.2143	-0.1433	0.1633	-0.0741	0.1503
		1000	-0.1347	0.1472	-0.0860	0.1312	-0.1218	0.1319	-0.0531	0.0937
	1.1	150	-0.2284	0.3079	-0.1971	0.4493	-0.2121	0.2957	-0.1282	0.5014
		300	-0.1659	0.2045	-0.1265	0.2475	-0.1621	0.1992	-0.0821	0.2287
		500	-0.1544	0.1829	-0.1318	0.2224	-0.1356	0.1571	-0.0685	0.1498
		1000	-0.1247	0.1386	-0.0829	0.1300	-0.1133	0.1245	-0.0496	0.0948
	1.3	150	-0.2151	0.3023	-0.1781	0.4384	-0.1985	0.2867	-0.1193	0.4914
		300	-0.1516	0.1958	-0.1152	0.2383	-0.1454	0.1863	-0.0755	0.2199
		500	-0.1481	0.1794	-0.1234	0.2138	-0.1252	0.1495	-0.0643	0.1511
		1000	-0.1165	0.1329	-0.0778	0.1275	-0.1033	0.1158	-0.0451	0.0937
	1.5	150	-0.2008	0.2964	-0.1806	0.4321	-0.1730	0.2675	-0.1137	0.4725
		300	-0.1400	0.1883	-0.1100	0.2337	-0.1316	0.1770	-0.0704	0.2169
		500	-0.1352	0.1702	-0.1204	0.2151	-0.1094	0.1376	-0.0615	0.1505
		1000	-0.1065	0.1248	-0.0754	0.1280	-0.0895	0.1044	-0.0411	0.0924
	1.7	150	-0.1704	0.2719	-0.1847	0.4359	-0.1523	0.2499	-0.1066	0.4309
		300	-0.1202	0.1740	-0.1094	0.2374	-0.1110	0.1609	-0.0618	0.2005
		500	-0.1209	0.1591	-0.1188	0.2161	-0.0921	0.1239	-0.0566	0.1469
		1000	-0.0918	0.1127	-0.0729	0.1281	-0.0758	0.0930	-0.0367	0.0895
	1.9	150	-0.1515	0.2582	-0.1801	0.4245	-0.1214	0.2201	-0.0861	0.3824
		300	-0.1065	0.1645	-0.1051	0.2326	-0.0895	0.1417	-0.0595	0.1925
		500	-0.1036	0.1509	-0.1131	0.2117	-0.0757	0.1133	-0.0511	0.1371
		1000	-0.0802	0.1046	-0.0666	0.1258	-0.0616	0.0832	-0.0313	0.0861

[a] The data generation is $(1 - .8L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;
[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 47: Monte Carlo Results of the Log-Regression Wavelet Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	-0.3558	0.4074	-0.2916	0.5002	-0.3677	0.4202	-0.2388	0.5517
		300	-0.2796	0.3014	-0.1975	0.2826	-0.2890	0.3082	-0.1742	0.2679
		500	-0.2626	0.2796	-0.1928	0.2555	-0.2511	0.2615	-0.1439	0.1908
		1000	-0.2228	0.2299	-0.1366	0.1654	-0.2182	0.2227	-0.1129	0.1351
	.7	150	-0.3500	0.4050	-0.2891	0.5001	-0.3596	0.4136	-0.2431	0.5408
		300	-0.2743	0.2984	-0.1870	0.2774	-0.2884	0.3082	-0.1635	0.2645
		500	-0.2564	0.2740	-0.1856	0.2501	-0.2439	0.2551	-0.1332	0.1847
		1000	-0.2168	0.2245	-0.1302	0.1604	-0.2118	0.2169	-0.1049	0.1292
	.9	150	-0.3383	0.3951	-0.2734	0.4954	-0.3527	0.4080	-0.2282	0.5386
		300	-0.2673	0.2919	-0.1773	0.2720	-0.2766	0.2988	-0.1531	0.2623
		500	-0.2486	0.2671	-0.1777	0.2462	-0.2318	0.2445	-0.1240	0.1789
		1000	-0.2091	0.2172	-0.1228	0.1563	-0.2040	0.2095	-0.0976	0.1250
	1.1	150	-0.3282	0.3903	-0.2655	0.4874	-0.3313	0.3900	-0.2149	0.5309
		300	-0.2531	0.2807	-0.1682	0.2684	-0.2608	0.2858	-0.1405	0.2543
		500	-0.2363	0.2567	-0.1703	0.2413	-0.2245	0.2388	-0.1159	0.1775
		1000	-0.1991	0.2079	-0.1176	0.1556	-0.1926	0.1993	-0.0906	0.1208
	1.3	150	-0.3097	0.3752	-0.2514	0.4717	-0.3130	0.3756	-0.1927	0.5236
		300	-0.2390	0.2687	-0.1642	0.2729	-0.2452	0.2728	-0.1336	0.2497
		500	-0.2228	0.2439	-0.1604	0.2377	-0.2071	0.2224	-0.1062	0.1724
		1000	-0.1855	0.1960	-0.1120	0.1511	-0.1780	0.1858	-0.0823	0.1172
	1.5	150	-0.2865	0.3574	-0.2274	0.4611	-0.2890	0.3575	-0.1752	0.4911
		300	-0.2216	0.2548	-0.1491	0.2583	-0.2233	0.2530	-0.1180	0.2383
		500	-0.2066	0.2306	-0.1529	0.2321	-0.1888	0.2067	-0.0939	0.1641
		1000	-0.1717	0.1832	-0.1026	0.1459	-0.1600	0.1690	-0.0713	0.1093
	1.7	150	-0.2597	0.3330	-0.2228	0.4518	-0.2509	0.3215	-0.1599	0.4634
		300	-0.1968	0.2336	-0.1388	0.2529	-0.1947	0.2283	-0.1008	0.2212
		500	-0.1838	0.2118	-0.1447	0.2293	-0.1645	0.1850	-0.0882	0.1599
		1000	-0.1521	0.1661	-0.0959	0.1428	-0.1409	0.1511	-0.0637	0.1043
	1.9	150	-0.2227	0.3102	-0.2009	0.4358	-0.2153	0.2910	-0.1348	0.4033
		300	-0.1643	0.2090	-0.1308	0.2494	-0.1604	0.1970	-0.0830	0.2009
		500	-0.1631	0.1971	-0.1376	0.2276	-0.1368	0.1608	-0.0682	0.1447
		1000	-0.1293	0.1467	-0.0859	0.1384	-0.1163	0.1295	-0.0545	0.1007

[a] The data generation is $(1-L)^d X_t = (1 - .4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 48: Monte Carlo Results of the Log-Regression Wavelet Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .4$

θ	d	n	K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
			bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	-0.0335	0.2026	-0.1307	0.4267	0.0377	0.2096	-0.0368	0.4890
		300	-0.0038	0.1140	-0.0709	0.2181	0.0351	0.1143	-0.0174	0.2061
		500	-0.0123	0.0949	-0.0851	0.1877	0.0274	0.0794	-0.0171	0.1255
		1000	0.0069	0.0577	-0.0448	0.1035	0.0307	0.0548	-0.0095	0.0741
	.7	150	-0.0421	0.2082	-0.1399	0.4380	0.0160	0.2038	-0.0384	0.4812
		300	-0.0144	0.1179	-0.0778	0.2227	0.0223	0.1127	-0.0266	0.2091
		500	-0.0238	0.1003	-0.0890	0.1913	0.0186	0.0777	-0.0236	0.1302
		1000	-0.0019	0.0587	-0.0470	0.1058	0.0199	0.0512	-0.0128	0.0771
	.9	150	-0.0552	0.2160	-0.1567	0.4352	0.0106	0.2070	-0.0463	0.4894
		300	-0.0236	0.1205	-0.0878	0.2272	0.0101	0.1136	-0.0293	0.2111
		500	-0.0315	0.1022	-0.0927	0.1951	0.0070	0.0781	-0.0255	0.1355
		1000	-0.0103	0.0602	-0.0565	0.1155	0.0117	0.0507	-0.0160	0.0803
	1.1	150	-0.0642	0.2175	-0.1503	0.4355	-0.0108	0.2086	-0.0522	0.4916
		300	-0.0303	0.1264	-0.0854	0.2256	-0.0005	0.1172	-0.0306	0.2108
		500	-0.0408	0.1087	-0.0973	0.1999	-0.0001	0.0801	-0.0315	0.1377
		1000	-0.0164	0.0622	-0.0563	0.1140	0.0040	0.0510	-0.0181	0.0832
	1.3	150	-0.0695	0.2255	-0.1508	0.4272	-0.0234	0.2122	-0.0602	0.4757
		300	-0.0415	0.1304	-0.0888	0.2294	-0.0107	0.1191	-0.0379	0.2160
		500	-0.0458	0.1112	-0.0999	0.2027	-0.0062	0.0808	-0.0304	0.1385
		1000	-0.0222	0.0652	-0.0565	0.1172	-0.0014	0.0529	-0.0207	0.0840
	1.5	150	-0.0801	0.2300	-0.1587	0.4320	-0.0305	0.2072	-0.0600	0.4471
		300	-0.0443	0.1331	-0.0948	0.2379	-0.0139	0.1180	-0.0363	0.2026
		500	-0.0510	0.1171	-0.1010	0.2053	-0.0129	0.0846	-0.0367	0.1415
		1000	-0.0269	0.0694	-0.0614	0.1196	-0.0054	0.0542	-0.0205	0.0843
	1.7	150	-0.0851	0.2269	-0.1540	0.4214	-0.0288	0.1932	-0.0606	0.4161
		300	-0.0462	0.1339	-0.0884	0.2287	-0.0201	0.1143	-0.0398	0.2045
		500	-0.0549	0.1198	-0.0999	0.2031	-0.0168	0.0843	-0.0325	0.1367
		1000	-0.0290	0.0713	-0.0571	0.1177	-0.0067	0.0542	-0.0227	0.0870
	1.9	150	-0.0820	0.2205	-0.1681	0.4211	-0.0293	0.1802	-0.0639	0.3713
		300	-0.0498	0.1327	-0.0884	0.2254	-0.0226	0.1096	-0.0344	0.1789
		500	-0.0585	0.1228	-0.0990	0.2066	-0.0203	0.0843	-0.0353	0.1326
		1000	-0.0309	0.0737	-0.0587	0.1243	-0.0102	0.0554	-0.0207	0.0841

[a] The data generation is $(1-L)^d X_t = (1 + .4L)\varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 49: Monte Carlo Results of the Log-Regression Wavelet Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$

θ	d	n	K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
			bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	-0.0072	0.2007	-0.1361	0.4244	0.0576	0.2143	-0.0153	0.4840
		300	0.0168	0.1131	-0.0625	0.2130	0.0548	0.1228	-0.0046	0.2034
		500	0.0010	0.0931	-0.0833	0.1848	0.0452	0.0864	-0.0154	0.1286
		1000	0.0197	0.0605	-0.0410	0.1014	0.0468	0.0655	-0.0051	0.0735
	.7	150	-0.0249	0.2053	-0.1405	0.4299	0.0430	0.2111	-0.0328	0.4893
		300	0.0022	0.1152	-0.0774	0.2184	0.0423	0.1183	-0.0198	0.2096
		500	-0.0114	0.0999	-0.0891	0.1922	0.0325	0.0817	-0.0195	0.1275
		1000	0.0107	0.0579	-0.0470	0.1089	0.0357	0.0593	-0.0094	0.0758
	.9	150	-0.0418	0.2124	-0.1492	0.4336	0.0242	0.2101	-0.0455	0.4875
		300	-0.0091	0.1182	-0.0820	0.2216	0.0230	0.1168	-0.0253	0.2149
		500	-0.0207	0.0991	-0.0920	0.1951	0.0229	0.0822	-0.0241	0.1354
		1000	0.0005	0.0598	-0.0502	0.1101	0.0245	0.0553	-0.0133	0.0792
	1.1	150	-0.0511	0.2174	-0.1496	0.4364	0.0061	0.2120	-0.0349	0.4759
		300	-0.0207	0.1234	-0.0790	0.2244	0.0109	0.1183	-0.0274	0.2123
		500	-0.0311	0.1068	-0.0916	0.1951	0.0115	0.0812	-0.0243	0.1358
		1000	-0.0060	0.0608	-0.0521	0.1112	0.0154	0.0531	-0.0137	0.0814
	1.3	150	-0.0649	0.2218	-0.1504	0.4316	-0.0015	0.2084	-0.0509	0.4802
		300	-0.0304	0.1276	-0.0803	0.2282	0.0016	0.1166	-0.0303	0.2107
		500	-0.0381	0.1103	-0.0937	0.1958	0.0036	0.0817	-0.0290	0.1389
		1000	-0.0138	0.0636	-0.0558	0.1176	0.0072	0.0529	-0.0177	0.0837
	1.5	150	-0.0726	0.2238	-0.1674	0.4442	-0.0172	0.2021	-0.0573	0.4562
		300	-0.0330	0.1265	-0.0901	0.2325	-0.0028	0.1155	-0.0388	0.2044
		500	-0.0458	0.1161	-0.0989	0.2009	-0.0034	0.0815	-0.0343	0.1397
		1000	-0.0192	0.0664	-0.0550	0.1156	0.0024	0.0532	-0.0201	0.0852
	1.7	150	-0.0762	0.2202	-0.1533	0.4232	-0.0208	0.1941	-0.0635	0.4176
		300	-0.0391	0.1303	-0.0858	0.2259	-0.0119	0.1139	-0.0439	0.2015
		500	-0.0475	0.1155	-0.0987	0.2014	-0.0098	0.0818	-0.0320	0.1345
		1000	-0.0245	0.0689	-0.0590	0.1207	-0.0027	0.0538	-0.0194	0.0837
	1.9	150	-0.0777	0.2187	-0.1519	0.4081	-0.0254	0.1782	-0.0610	0.3705
		300	-0.0432	0.1318	-0.0822	0.2198	-0.0161	0.1098	-0.0373	0.1823
		500	-0.0518	0.1172	-0.0990	0.2037	-0.0132	0.0821	-0.0294	0.1287
		1000	-0.0261	0.0714	-0.0561	0.1194	-0.0061	0.0553	-0.0190	0.0836

[a] The data generation is $(1-L)^d X_t = (1+.8L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 50: Monte Carlo Results the Local Whittle Wavelet Estimation for ARFIMA$(0, d, 0)$ Model

		K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.5	150	-0.0431	0.1748	-0.0666	0.3538	-0.0419	0.2003	-0.0764	0.4907
	300	-0.0305	0.0996	-0.0239	0.1691	-0.0333	0.1109	-0.0245	0.2046
	500	-0.0260	0.0718	-0.0157	0.1123	-0.0269	0.0741	-0.0182	0.1234
	1000	-0.0220	0.0488	-0.0112	0.0675	-0.0211	0.0483	-0.0095	0.0704
.7	150	-0.0529	0.1837	-0.0581	0.3509	-0.0612	0.2149	-0.0768	0.5040
	300	-0.0368	0.1053	-0.0314	0.1757	-0.0407	0.1155	-0.0307	0.2085
	500	-0.0308	0.0765	-0.0183	0.1175	-0.0330	0.0781	-0.0194	0.1224
	1000	-0.0250	0.0509	-0.0112	0.0704	-0.0253	0.0518	-0.0118	0.0735
.9	150	-0.0608	0.1870	-0.0668	0.3568	-0.0662	0.2141	-0.0912	0.5023
	300	-0.0413	0.1114	-0.0308	0.1801	-0.0427	0.1188	-0.0350	0.2080
	500	-0.0319	0.0791	-0.0218	0.1202	-0.0342	0.0827	-0.0214	0.1279
	1000	-0.0276	0.0552	-0.0124	0.0735	-0.0288	0.0553	-0.0128	0.0760
1.1	150	-0.0599	0.1871	-0.0743	0.3663	-0.0595	0.2093	-0.0934	0.4843
	300	-0.0445	0.1137	-0.0345	0.1827	-0.0457	0.1216	-0.0369	0.2130
	500	-0.0357	0.0827	-0.0234	0.1252	-0.0356	0.0847	-0.0207	0.1297
	1000	-0.0292	0.0571	-0.0123	0.0749	-0.0307	0.0574	-0.0134	0.0790
1.3	150	-0.0658	0.1894	-0.0743	0.3531	-0.0714	0.2176	-0.0777	0.4755
	300	-0.0422	0.1144	-0.0369	0.1836	-0.0451	0.1238	-0.0369	0.2042
	500	-0.0373	0.0850	-0.0246	0.1278	-0.0379	0.0865	-0.0231	0.1312
	1000	-0.0297	0.0580	-0.0159	0.0781	-0.0299	0.0590	-0.0148	0.0792
1.5	150	-0.0643	0.1896	-0.0788	0.3457	-0.0670	0.2096	-0.0844	0.4545
	300	-0.0431	0.1154	-0.0391	0.1822	-0.0441	0.1202	-0.0355	0.2031
	500	-0.0359	0.0854	-0.0228	0.1259	-0.0362	0.0867	-0.0234	0.1279
	1000	-0.0271	0.0581	-0.0140	0.0779	-0.0295	0.0590	-0.0134	0.0803
1.7	150	-0.0645	0.1857	-0.0859	0.3384	-0.0664	0.2032	-0.0819	0.4247
	300	-0.0425	0.1144	-0.0399	0.1787	-0.0407	0.1188	-0.0373	0.1913
	500	-0.0329	0.0841	-0.0243	0.1241	-0.0339	0.0849	-0.0245	0.1255
	1000	-0.0254	0.0568	-0.0146	0.0788	-0.0280	0.0588	-0.0162	0.0791
1.9	150	-0.0606	0.1775	-0.0930	0.3219	-0.0550	0.1838	-0.0555	0.3736
	300	-0.0390	0.1106	-0.0440	0.1737	-0.0390	0.1136	-0.0388	0.1784
	500	-0.0310	0.0840	-0.0292	0.1208	-0.0322	0.0835	-0.0259	0.1192
	1000	-0.0235	0.0576	-0.0175	0.0790	-0.0241	0.0577	-0.0150	0.0782

[a] The data generation is $(1-L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 51: Monte Carlo Results the Local Whittle Wavelet Estimation for ARFIMA$(1, d, 0)$ Model with $\phi = -.4$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	0.1875	0.2556	0.0667	0.3545	0.2052	0.2891	0.0728	0.4926
		300	0.1669	0.1949	0.0751	0.1855	0.1815	0.2096	0.0864	0.2195
		500	0.1604	0.1747	0.0708	0.1331	0.1629	0.1781	0.0764	0.1434
		1000	0.1473	0.1536	0.0643	0.0930	0.1493	0.1561	0.0683	0.0985
	.7	150	0.1643	0.2383	0.0456	0.3611	0.1848	0.2716	0.0563	0.4944
		300	0.1507	0.1816	0.0591	0.1856	0.1613	0.1938	0.0721	0.2152
		500	0.1384	0.1555	0.0598	0.1310	0.1462	0.1635	0.0648	0.1416
		1000	0.1299	0.1377	0.0585	0.0919	0.1339	0.1420	0.0596	0.0940
	.9	150	0.1395	0.2234	0.0325	0.3535	0.1670	0.2617	0.0380	0.4870
		300	0.1307	0.1665	0.0500	0.1889	0.1421	0.1802	0.0548	0.2151
		500	0.1262	0.1454	0.0489	0.1299	0.1301	0.1497	0.0558	0.1366
		1000	0.1165	0.1255	0.0475	0.0864	0.1189	0.1282	0.0526	0.0923
	1.1	150	0.1173	0.2110	0.0169	0.3568	0.1394	0.2435	0.0264	0.4741
		300	0.1121	0.1529	0.0397	0.1818	0.1225	0.1665	0.0434	0.2131
		500	0.1075	0.1314	0.0377	0.1261	0.1125	0.1362	0.0433	0.1363
		1000	0.1009	0.1120	0.0399	0.0851	0.1034	0.1148	0.0418	0.0883
	1.3	150	0.0971	0.2017	-0.0015	0.3317	0.1138	0.2304	0.0031	0.4607
		300	0.0952	0.1438	0.0288	0.1785	0.1019	0.1501	0.0403	0.2083
		500	0.0912	0.1181	0.0289	0.1256	0.0957	0.1228	0.0302	0.1330
		1000	0.0855	0.0984	0.0328	0.0829	0.0898	0.1026	0.0346	0.0865
	1.5	150	0.0765	0.1869	-0.0057	0.3324	0.0916	0.2088	-0.0010	0.4333
		300	0.0795	0.1313	0.0167	0.1749	0.0874	0.1401	0.0206	0.1992
		500	0.0757	0.1066	0.0201	0.1197	0.0795	0.1106	0.0236	0.1271
		1000	0.0726	0.0881	0.0249	0.0799	0.0732	0.0886	0.0272	0.0832
	1.7	150	0.0552	0.1761	-0.0361	0.3179	0.0686	0.1929	-0.0184	0.4093
		300	0.0577	0.1180	0.0005	0.1691	0.0666	0.1244	0.0114	0.1850
		500	0.0587	0.0947	0.0135	0.1182	0.0631	0.0980	0.0170	0.1214
		1000	0.0562	0.0749	0.0172	0.0772	0.0586	0.0765	0.0199	0.0788
	1.9	150	0.0315	0.1579	-0.0423	0.2997	0.0474	0.1690	-0.0217	0.3555
		300	0.0380	0.1065	-0.0133	0.1625	0.0472	0.1104	0.0041	0.1658
		500	0.0395	0.0833	0.0009	0.1151	0.0449	0.0852	0.0059	0.1160
		1000	0.0399	0.0640	0.0072	0.0755	0.0427	0.0653	0.0103	0.0765

[a] The data generation is $(1 + .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 52: Monte Carlo Results the Local Whittle Wavelet Estimation for ARFIMA$(1, d, 0)$ Model $\phi = .4$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	-0.1537	0.2286	-0.0998	0.3643	-0.1823	0.2703	-0.1401	0.5170
		300	-0.1224	0.1552	-0.0577	0.1818	-0.1392	0.1754	-0.0665	0.2151
		500	-0.1104	0.1292	-0.0432	0.1209	-0.1139	0.1339	-0.0484	0.1305
		1000	-0.0968	0.1061	-0.0332	0.0753	-0.0988	0.1079	-0.0355	0.0784
	.7	150	-0.1554	0.2341	-0.0995	0.3599	-0.1780	0.2684	-0.1244	0.5045
		300	-0.1220	0.1569	-0.0568	0.1832	-0.1334	0.1721	-0.0599	0.2111
		500	-0.1077	0.1283	-0.0442	0.1256	-0.1115	0.1326	-0.0414	0.1287
		1000	-0.0935	0.1036	-0.0309	0.0752	-0.0964	0.1068	-0.0338	0.0787
	.9	150	-0.1538	0.2342	-0.0878	0.3592	-0.1743	0.2674	-0.1132	0.4960
		300	-0.1173	0.1565	-0.0518	0.1845	-0.1278	0.1687	-0.0629	0.2190
		500	-0.1012	0.1245	-0.0387	0.1259	-0.1080	0.1304	-0.0402	0.1328
		1000	-0.0891	0.1012	-0.0305	0.0793	-0.0917	0.1032	-0.0312	0.0802
	1.1	150	-0.1423	0.2278	-0.0972	0.3679	-0.1681	0.2663	-0.1048	0.4889
		300	-0.1121	0.1532	-0.0540	0.1886	-0.1184	0.1637	-0.0529	0.2146
		500	-0.0974	0.1228	-0.0380	0.1294	-0.1023	0.1279	-0.0410	0.1336
		1000	-0.0837	0.0971	-0.0288	0.0805	-0.0858	0.0990	-0.0272	0.0820
	1.3	150	-0.1370	0.2254	-0.0932	0.3589	-0.1499	0.2533	-0.1067	0.4909
		300	-0.1009	0.1468	-0.0532	0.1889	-0.1121	0.1597	-0.0560	0.2143
		500	-0.0907	0.1188	-0.0370	0.1302	-0.0926	0.1213	-0.0370	0.1323
		1000	-0.0769	0.0918	-0.0271	0.0810	-0.0775	0.0927	-0.0274	0.0830
	1.5	150	-0.1241	0.2192	-0.0958	0.3534	-0.1420	0.2462	-0.1030	0.4660
		300	-0.0930	0.1423	-0.0452	0.1823	-0.1014	0.1519	-0.0481	0.2071
		500	-0.0804	0.1121	-0.0374	0.1294	-0.0820	0.1147	-0.0369	0.1326
		1000	-0.0670	0.0845	-0.0236	0.0812	-0.0693	0.0861	-0.0241	0.0816
	1.7	150	-0.1128	0.2095	-0.0901	0.3383	-0.1193	0.2283	-0.0937	0.4340
		300	-0.0819	0.1353	-0.0462	0.1808	-0.0877	0.1427	-0.0435	0.1934
		500	-0.0690	0.1043	-0.0343	0.1267	-0.0713	0.1070	-0.0353	0.1284
		1000	-0.0574	0.0772	-0.0213	0.0812	-0.0609	0.0808	-0.0247	0.0820
	1.9	150	-0.0970	0.1963	-0.0978	0.3243	-0.1020	0.2079	-0.0766	0.3838
		300	-0.0690	0.1261	-0.0511	0.1712	-0.0712	0.1294	-0.0452	0.1809
		500	-0.0586	0.0987	-0.0349	0.1231	-0.0594	0.0990	-0.0291	0.1208
		1000	-0.0479	0.0718	-0.0217	0.0797	-0.0481	0.0711	-0.0228	0.0802

[a] The data generation is $(1 - .4L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 53: Monte Carlo Results the Local Whittle Wavelet Estimation for ARFIMA$(1, d, 0)$ Model $\phi = .8$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
ϕ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	-0.1942	0.2571	-0.1175	0.3731	-0.2257	0.2989	-0.1547	0.5185
		300	-0.1554	0.1831	-0.0748	0.1851	-0.1716	0.2027	-0.0869	0.2182
		500	-0.1401	0.1553	-0.0620	0.1301	-0.1469	0.1627	-0.0674	0.1385
		1000	-0.1215	0.1288	-0.0473	0.0820	-0.1274	0.1349	-0.0511	0.0858
	.7	150	-0.1934	0.2578	-0.1109	0.3720	-0.2186	0.2958	-0.1358	0.5001
		300	-0.1508	0.1800	-0.0653	0.1844	-0.1672	0.1998	-0.0759	0.2156
		500	-0.1356	0.1527	-0.0568	0.1299	-0.1408	0.1578	-0.0558	0.1354
		1000	-0.1179	0.1260	-0.0421	0.0818	-0.1208	0.1292	-0.0406	0.0817
	.9	150	-0.1838	0.2531	-0.1170	0.3753	-0.2101	0.2911	-0.1294	0.5003
		300	-0.1450	0.1787	-0.0608	0.1851	-0.1600	0.1939	-0.0614	0.2147
		500	-0.1270	0.1461	-0.0473	0.1275	-0.1324	0.1524	-0.0514	0.1375
		1000	-0.1106	0.1200	-0.0387	0.0822	-0.1132	0.1230	-0.0362	0.0833
	1.1	150	-0.1719	0.2477	-0.0993	0.3641	-0.1971	0.2849	-0.1282	0.5014
		300	-0.1364	0.1729	-0.0545	0.1847	-0.1472	0.1862	-0.0620	0.2177
		500	-0.1186	0.1399	-0.0449	0.1290	-0.1229	0.1452	-0.0468	0.1359
		1000	-0.1006	0.1115	-0.0320	0.0803	-0.1053	0.1163	-0.0339	0.0835
	1.3	150	-0.1542	0.2370	-0.0904	0.3560	-0.1818	0.2772	-0.1193	0.4914
		300	-0.1222	0.1618	-0.0520	0.1887	-0.1327	0.1773	-0.0570	0.2099
		500	-0.1047	0.1288	-0.0402	0.1280	-0.1104	0.1355	-0.0400	0.1324
		1000	-0.0923	0.1051	-0.0307	0.0837	-0.0942	0.1071	-0.0288	0.0823
	1.5	150	-0.1461	0.2353	-0.0987	0.3537	-0.1615	0.2573	-0.1137	0.4725
		300	-0.1085	0.1541	-0.0554	0.1889	-0.1177	0.1646	-0.0532	0.2070
		500	-0.0960	0.1241	-0.0398	0.1281	-0.0979	0.1264	-0.0407	0.1327
		1000	-0.0800	0.0953	-0.0251	0.0810	-0.0818	0.0969	-0.0270	0.0826
	1.7	150	-0.1221	0.2123	-0.0916	0.3401	-0.1344	0.2342	-0.1066	0.4309
		300	-0.0928	0.1422	-0.0548	0.1810	-0.1013	0.1515	-0.0483	0.1920
		500	-0.0790	0.1119	-0.0376	0.1276	-0.0839	0.1161	-0.0347	0.1271
		1000	-0.0683	0.0861	-0.0247	0.0811	-0.0704	0.0879	-0.0243	0.0818
	1.9	150	-0.1109	0.2052	-0.0911	0.3152	-0.1147	0.2186	-0.0861	0.3824
		300	-0.0823	0.1357	-0.0472	0.1730	-0.0827	0.1347	-0.0492	0.1835
		500	-0.0679	0.1030	-0.0337	0.1211	-0.0703	0.1050	-0.0332	0.1241
		1000	-0.0561	0.0772	-0.0217	0.0783	-0.0560	0.0764	-0.0237	0.0787

[a] The data generation is $(1 - .8L)(1 - L)^d X_t = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 54: Monte Carlo Results the Local Whittle Wavelet Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = -.4$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
-.4	.5	150	-0.3052	0.3488	-0.2001	0.4090	-0.3421	0.3942	-0.2450	0.5413
		300	-0.2539	0.2719	-0.1376	0.2214	-0.2756	0.2946	-0.1562	0.2573
		500	-0.2290	0.2393	-0.1152	0.1617	-0.2389	0.2490	-0.1234	0.1721
		1000	-0.2015	0.2064	-0.0950	0.1159	-0.2093	0.2141	-0.0991	0.1212
	.7	150	-0.3067	0.3528	-0.1966	0.4008	-0.3441	0.3990	-0.2372	0.5350
		300	-0.2481	0.2674	-0.1297	0.2164	-0.2733	0.2935	-0.1484	0.2527
		500	-0.2212	0.2320	-0.1059	0.1563	-0.2339	0.2448	-0.1126	0.1659
		1000	-0.1969	0.2023	-0.0865	0.1117	-0.2035	0.2086	-0.0907	0.1161
	.9	150	-0.2948	0.3437	-0.1820	0.3989	-0.3361	0.3924	-0.2331	0.5478
		300	-0.2394	0.2606	-0.1209	0.2137	-0.2636	0.2866	-0.1410	0.2498
		500	-0.2149	0.2269	-0.0978	0.1545	-0.2233	0.2357	-0.1062	0.1659
		1000	-0.1902	0.1958	-0.0803	0.1084	-0.1948	0.2011	-0.0836	0.1116
	1.1	150	-0.2803	0.3322	-0.1695	0.3903	-0.3195	0.3785	-0.2121	0.5375
		300	-0.2243	0.2483	-0.1094	0.2116	-0.2513	0.2756	-0.1217	0.2399
		500	-0.2024	0.2161	-0.0880	0.1518	-0.2136	0.2270	-0.0984	0.1610
		1000	-0.1793	0.1860	-0.0742	0.1062	-0.1844	0.1913	-0.0739	0.1076
	1.3	150	-0.2616	0.3180	-0.1676	0.3940	-0.3043	0.3665	-0.2038	0.5248
		300	-0.2097	0.2361	-0.1024	0.2067	-0.2315	0.2581	-0.1105	0.2339
		500	-0.1891	0.2041	-0.0804	0.1471	-0.1993	0.2145	-0.0888	0.1574
		1000	-0.1654	0.1728	-0.0647	0.1003	-0.1716	0.1791	-0.0678	0.1027
	1.5	150	-0.2413	0.3038	-0.1468	0.3753	-0.2733	0.3401	-0.1634	0.4880
		300	-0.1949	0.2241	-0.0944	0.2026	-0.2129	0.2425	-0.1035	0.2287
		500	-0.1705	0.1881	-0.0738	0.1449	-0.1792	0.1965	-0.0762	0.1514
		1000	-0.1484	0.1574	-0.0588	0.0972	-0.1535	0.1623	-0.0607	0.1003
	1.7	150	-0.2130	0.2816	-0.1288	0.3492	-0.2440	0.3136	-0.1524	0.4472
		300	-0.1733	0.2056	-0.0865	0.1978	-0.1852	0.2181	-0.0897	0.2159
		500	-0.1493	0.1695	-0.0661	0.1380	-0.1556	0.1753	-0.0684	0.1414
		1000	-0.1316	0.1418	-0.0515	0.0939	-0.1336	0.1437	-0.0512	0.0946
	1.9	150	-0.1841	0.2575	-0.1275	0.3393	-0.2016	0.2738	-0.1290	0.4021
		300	-0.1464	0.1833	-0.0753	0.1861	-0.1538	0.1888	-0.0797	0.1967
		500	-0.1276	0.1513	-0.0606	0.1350	-0.1302	0.1538	-0.0581	0.1326
		1000	-0.1100	0.1232	-0.0434	0.0886	-0.1117	0.1241	-0.0441	0.0894

[a] The data generation is $(1-L)^d X_t = (1 - .4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 55: Monte Carlo Results the Local Whittle Wavelet Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .4$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.4	.5	150	0.0536	0.1785	-0.0209	0.3529	0.0661	0.2084	-0.0276	0.4889
		300	0.0494	0.1072	-0.0018	0.1717	0.0554	0.1180	0.0006	0.2013
		500	0.0477	0.0827	0.0042	0.1144	0.0501	0.0862	0.0054	0.1210
		1000	0.0436	0.0618	0.0083	0.0675	0.0447	0.0632	0.0091	0.0698
	.7	150	0.0354	0.1803	-0.0282	0.3424	0.0447	0.2080	-0.0355	0.4813
		300	0.0368	0.1063	-0.0069	0.1725	0.0415	0.1159	-0.0052	0.2029
		500	0.0357	0.0792	0.0031	0.1164	0.0371	0.0821	0.0033	0.1233
		1000	0.0346	0.0563	0.0082	0.0700	0.0355	0.0575	0.0091	0.0725
	.9	150	0.0183	0.1783	-0.0382	0.3536	0.0277	0.2059	-0.0478	0.4919
		300	0.0237	0.1065	-0.0068	0.1783	0.0271	0.1156	-0.0027	0.2073
		500	0.0276	0.0778	-0.0031	0.1212	0.0287	0.0807	-0.0021	0.1284
		1000	0.0257	0.0542	0.0038	0.0728	0.0266	0.0554	0.0042	0.0752
	1.1	150	0.0099	0.1775	-0.0512	0.3622	0.0156	0.2028	-0.0541	0.4863
		300	0.0132	0.1059	-0.0150	0.1830	0.0165	0.1145	-0.0109	0.2080
		500	0.0172	0.0768	-0.0080	0.1219	0.0183	0.0788	-0.0070	0.1274
		1000	0.0175	0.0521	-0.0007	0.0748	0.0182	0.0531	-0.0003	0.0766
	1.3	150	-0.0055	0.1767	-0.0571	0.3519	-0.0012	0.1987	-0.0678	0.4772
		300	0.0075	0.1063	-0.0201	0.1785	0.0096	0.1136	-0.0171	0.2010
		500	0.0080	0.0767	-0.0078	0.1213	0.0090	0.0785	-0.0071	0.1270
		1000	0.0107	0.0508	-0.0017	0.0755	0.0111	0.0516	-0.0010	0.0769
	1.5	150	-0.0142	0.1771	-0.0618	0.3461	-0.0072	0.1928	-0.0664	0.4576
		300	-0.0009	0.1065	-0.0256	0.1786	0.0008	0.1133	-0.0216	0.1989
		500	0.0019	0.0769	-0.0162	0.1229	0.0029	0.0785	-0.0150	0.1267
		1000	0.0069	0.0518	-0.0056	0.0766	0.0073	0.0526	-0.0047	0.0781
	1.7	150	-0.0241	0.1734	-0.0803	0.3325	-0.0178	0.1856	-0.0701	0.4154
		300	-0.0094	0.1052	-0.0293	0.1722	-0.0069	0.1096	-0.0242	0.1857
		500	-0.0025	0.0765	-0.0159	0.1202	-0.0013	0.0773	-0.0140	0.1227
		1000	0.0018	0.0502	-0.0078	0.0772	0.0022	0.0504	-0.0070	0.0776
	1.9	150	-0.0301	0.1658	-0.0748	0.3144	-0.0208	0.1682	-0.0597	0.3705
		300	-0.0137	0.1026	-0.0366	0.1708	-0.0106	0.1019	-0.0307	0.1773
		500	-0.0077	0.0772	-0.0209	0.1198	-0.0062	0.0769	-0.0176	0.1190
		1000	-0.0029	0.0519	-0.0114	0.0755	-0.0025	0.0515	-0.0106	0.0751

[a] The data generation is $(1-L)^d X_t = (1+.4L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;

[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

Table 56: Monte Carlo Results of the Local Whittle Wavelet Estimation for ARFIMA$(0, d, 1)$ Model with $\theta = .8$

			K=0 J=1		K=0 J=2		K=1 J=1		K=1 J=2	
θ	d	n	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE
.8	.5	150	0.0730	0.1879	-0.0193	0.3500	0.0877	0.2180	-0.0316	0.4914
		300	0.0714	0.1217	-0.0000	0.1720	0.0773	0.1321	0.0130	0.2025
		500	0.0667	0.0947	0.0141	0.1152	0.0698	0.0988	0.0153	0.1217
		1000	0.0624	0.0765	0.0149	0.0699	0.0619	0.0759	0.0160	0.0722
	.7	150	0.0569	0.1852	-0.0327	0.3517	0.0718	0.2155	-0.0401	0.4931
		300	0.0553	0.1137	-0.0020	0.1734	0.0609	0.1264	0.0042	0.2066
		500	0.0516	0.0871	0.0034	0.1175	0.0548	0.0903	0.0067	0.1235
		1000	0.0504	0.0674	0.0111	0.0707	0.0507	0.0678	0.0111	0.0738
	.9	150	0.0418	0.1820	-0.0455	0.3522	0.0532	0.2130	-0.0562	0.4976
		300	0.0425	0.1117	-0.0060	0.1797	0.0486	0.1216	-0.0006	0.2035
		500	0.0401	0.0837	-0.0011	0.1192	0.0422	0.0861	0.0008	0.1276
		1000	0.0389	0.0608	0.0060	0.0730	0.0398	0.0615	0.0073	0.0736
	1.1	150	0.0227	0.1787	-0.0452	0.3519	0.0376	0.2072	-0.0612	0.4856
		300	0.0271	0.1103	-0.0117	0.1773	0.0343	0.1182	-0.0091	0.2038
		500	0.0305	0.0819	-0.0051	0.1216	0.0316	0.0841	-0.0014	0.1261
		1000	0.0294	0.0563	0.0022	0.0741	0.0308	0.0575	0.0027	0.0776
	1.3	150	0.0140	0.1760	-0.0543	0.3510	0.0138	0.2031	-0.0597	0.4814
		300	0.0155	0.1078	-0.0194	0.1819	0.0223	0.1155	-0.0071	0.2017
		500	0.0191	0.0787	-0.0070	0.1226	0.0223	0.0825	-0.0066	0.1312
		1000	0.0202	0.0538	0.0007	0.0761	0.0220	0.0560	-0.0002	0.0792
	1.5	150	-0.0026	0.1755	-0.0653	0.3425	0.0035	0.1950	-0.0661	0.4519
		300	0.0097	0.1059	-0.0220	0.1788	0.0109	0.1154	-0.0210	0.1948
		500	0.0107	0.0781	-0.0107	0.1249	0.0143	0.0810	-0.0124	0.1291
		1000	0.0129	0.0517	-0.0032	0.0764	0.0152	0.0538	-0.0027	0.0790
	1.7	150	-0.0142	0.1693	-0.0708	0.3330	-0.0055	0.1829	-0.0561	0.4151
		300	0.0001	0.1057	-0.0285	0.1753	0.0040	0.1084	-0.0209	0.1877
		500	0.0040	0.0745	-0.0139	0.1225	0.0072	0.0767	-0.0155	0.1206
		1000	0.0084	0.0524	-0.0079	0.0781	0.0072	0.0511	-0.0067	0.0772
	1.9	150	-0.0246	0.1665	-0.0755	0.3098	-0.0126	0.1704	-0.0620	0.3783
		300	-0.0108	0.1024	-0.0294	0.1686	-0.0044	0.1052	-0.0282	0.1748
		500	-0.0027	0.0753	-0.0211	0.1190	-0.0004	0.0767	-0.0193	0.1211
		1000	0.0020	0.0515	-0.0092	0.0765	0.0015	0.0515	-0.0106	0.0771

[a] The data generation is $(1-L)^d X_t = (1+.8L)\varepsilon_t$, $\varepsilon_t \sim N(0,1)$;
[b] The K is the trimming number of the highest wavelet scales; and the J is the trimming number of the lowest wavelet scales.

7.2 Figures

The figures concerned with the Monte Carlo simulation results are included here.

Figure 1: The fextLPWF Estimator Performance for the ARFIMA$(1, d, 0)$ Model with $d = .8$ and $\phi = -.4$

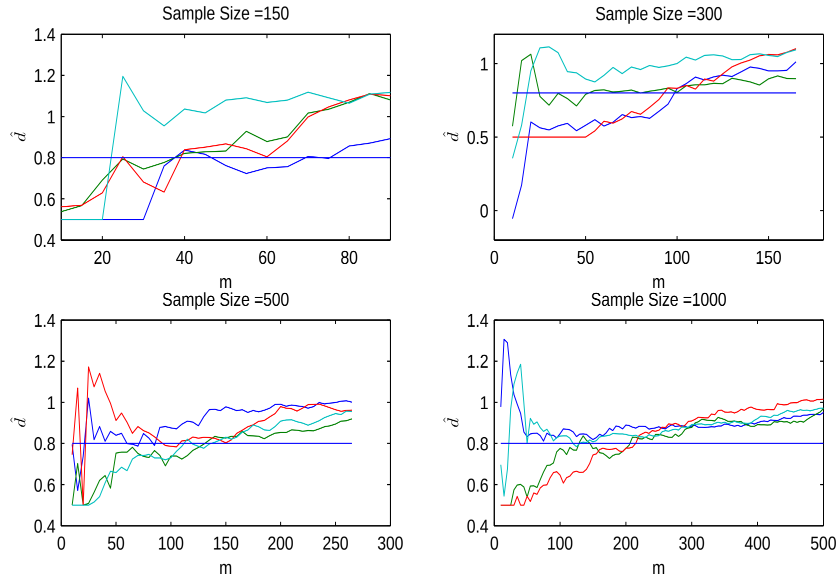


Figure 2: The fextLPWF Estimator Performance for the ARFIMA$(1, d, 0)$ Model with $d = .8$ and $\phi = .4$

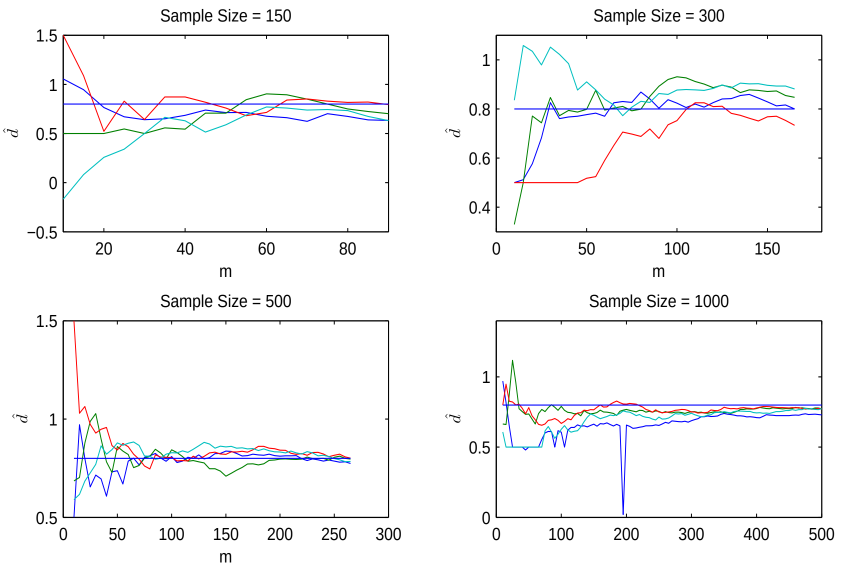


Figure 3: The fextLPWF Estimators' Performance for the ARFIMA$(0, d, 1)$ Model with $d = .8$ and $\theta = -.4$

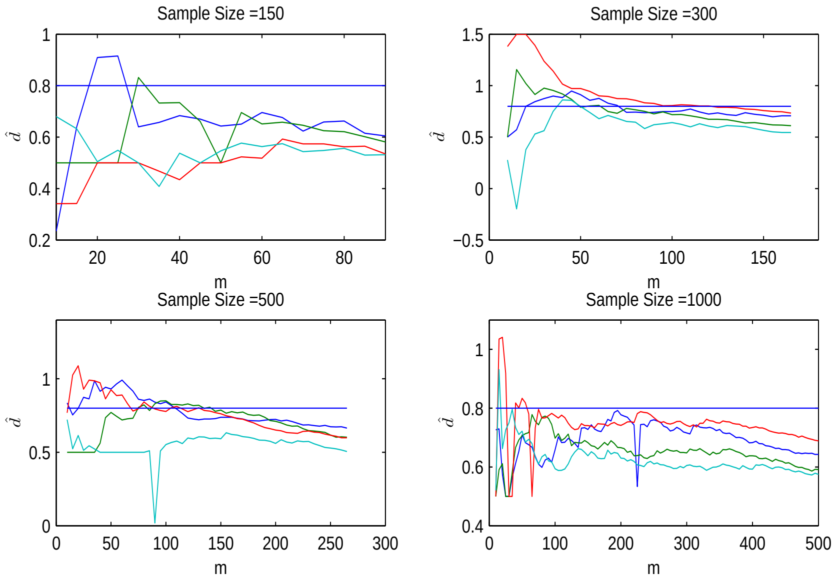


Figure 4: The fextLWF Estimator Performance for the ARFIMA$(1, d, 0)$ Model with $d = .8$ and $\phi = -.4$

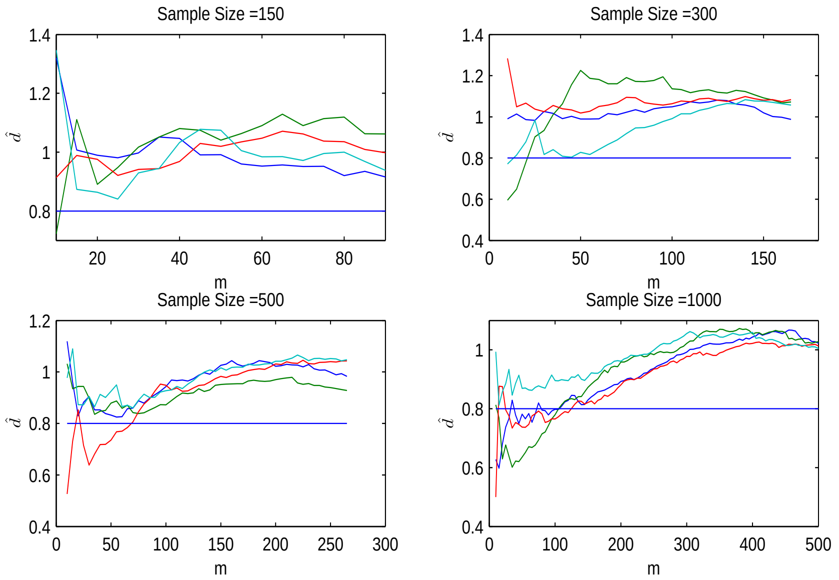


Figure 5: The fextLWF Estimator Performance for the ARFIMA$(1, d, 0)$ Model with $d = .8$ and $\phi = .4$

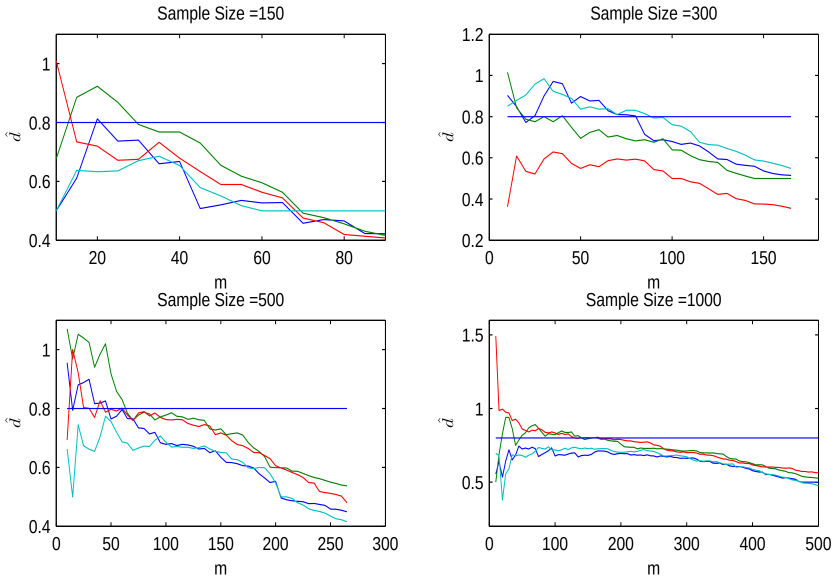


Figure 6: The fextLWF Estimators' Performance for the ARFIMA$(0, d, 1)$ Model with $d = .8$ and $\theta = -.4$

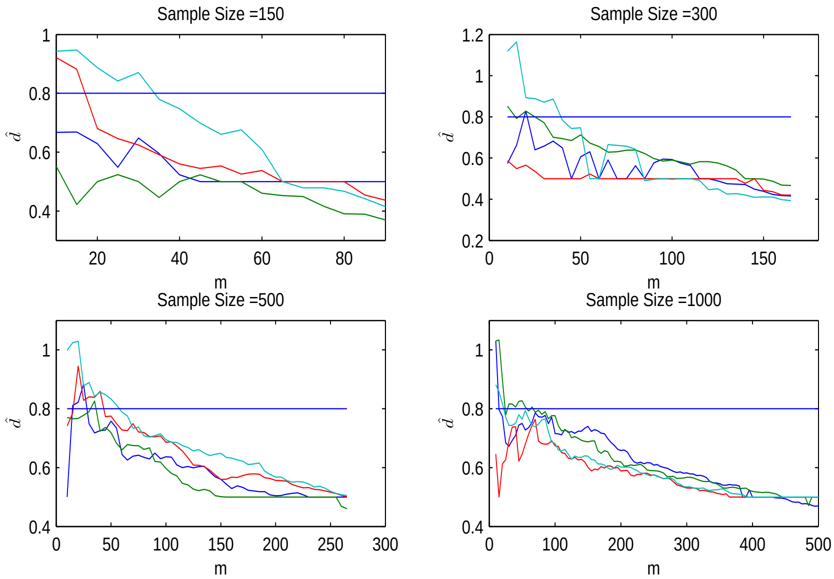


Figure 7: The feLWF Estimators' Performance for the ARFIMA$(1, d, 0)$ Model with $d = .8$ and $\phi = -.4$

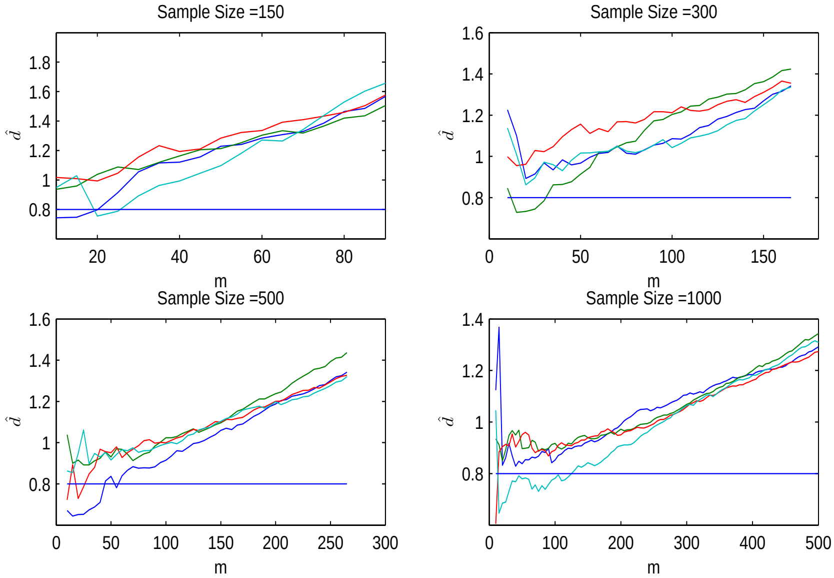


Figure 8: The feLWF Estimators' Performance for the ARFIMA$(0, d, 1)$ Model with $d = .8$ and $\theta = .4$

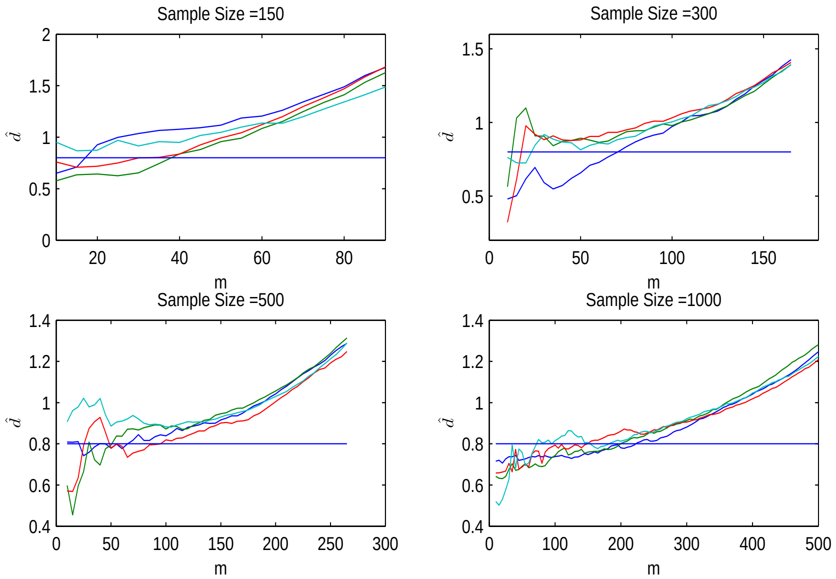


Figure 9: The modLWF Estimators' Performance for the ARFIMA$(1, d, 0)$ Model with $d = .8$ and $\phi = .8$

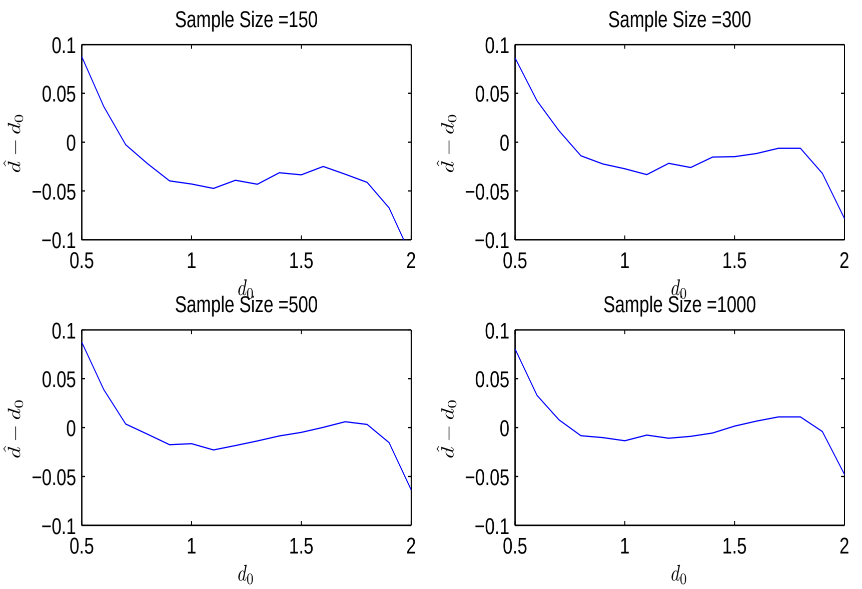


Figure 10: Boxplot of LRW Estimators for the ARFIMA$(1, d, 0)$ Model with $d = 1.2$ and $\theta = .4$

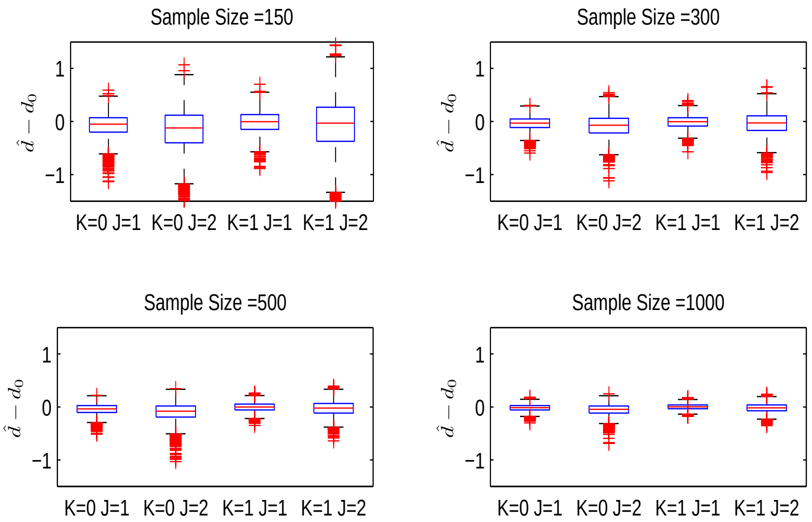
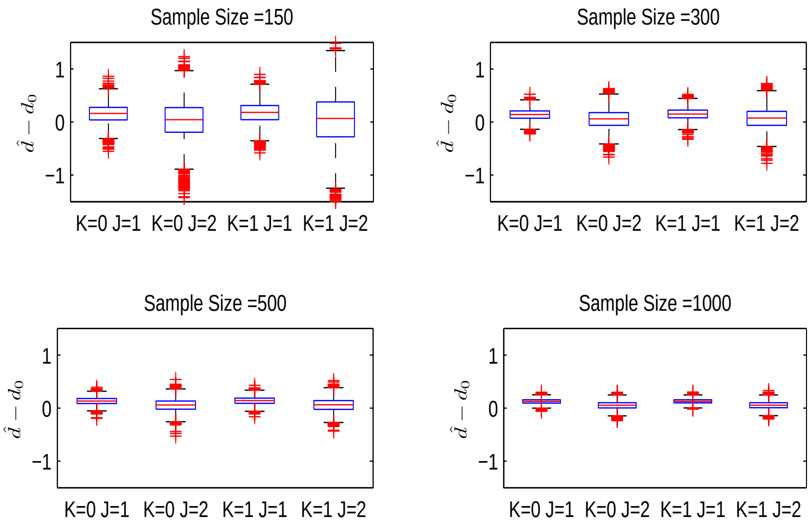


Figure 11: Boxplot of LWW Estimators for the ARFIMA$(1, d, 0)$ Model with $d = .8$ and $\phi = -.4$



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